GPU programming basics

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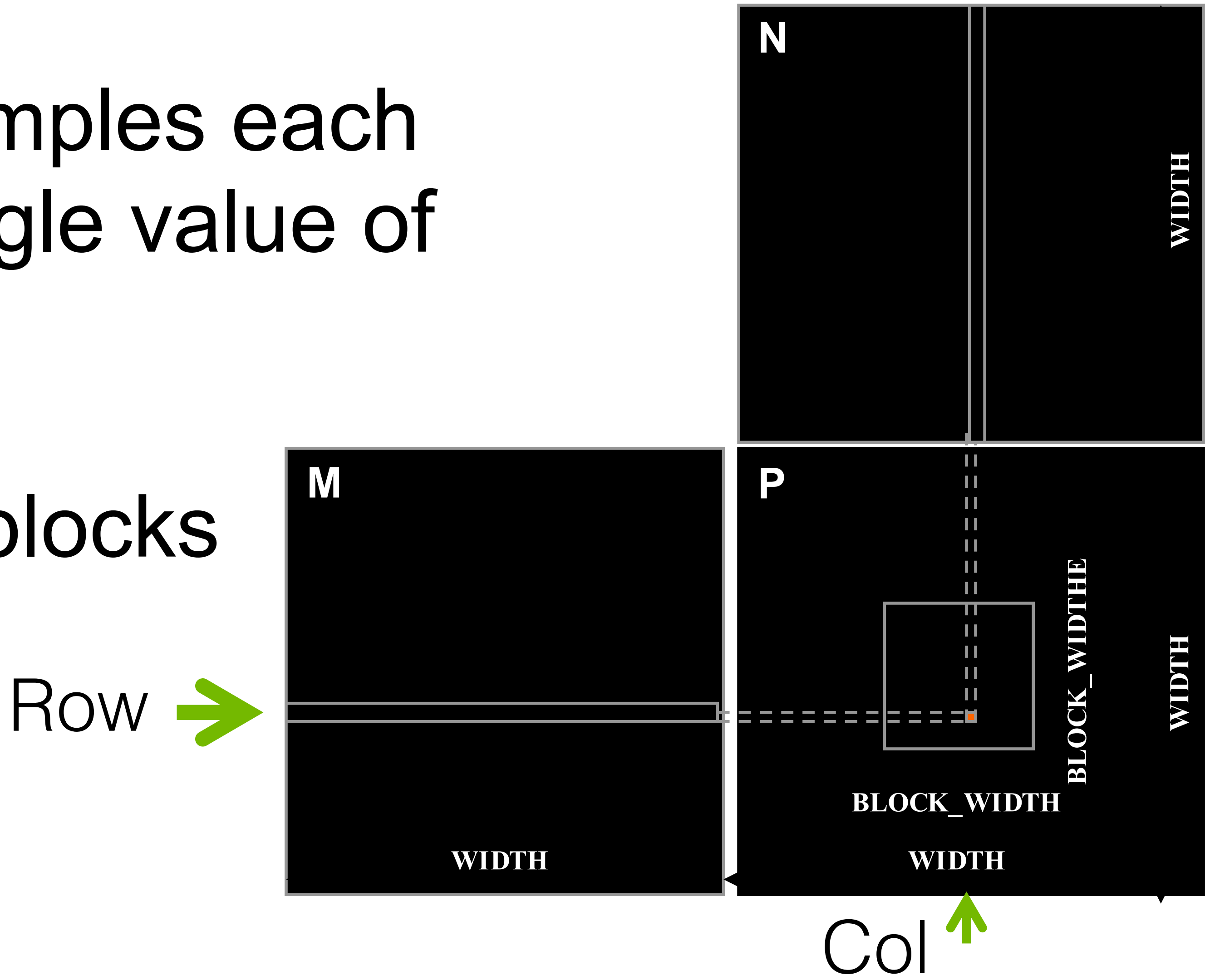
Data parallelism: GPU computing

CUDA: efficient programming

Basic problem: matrix multiplication

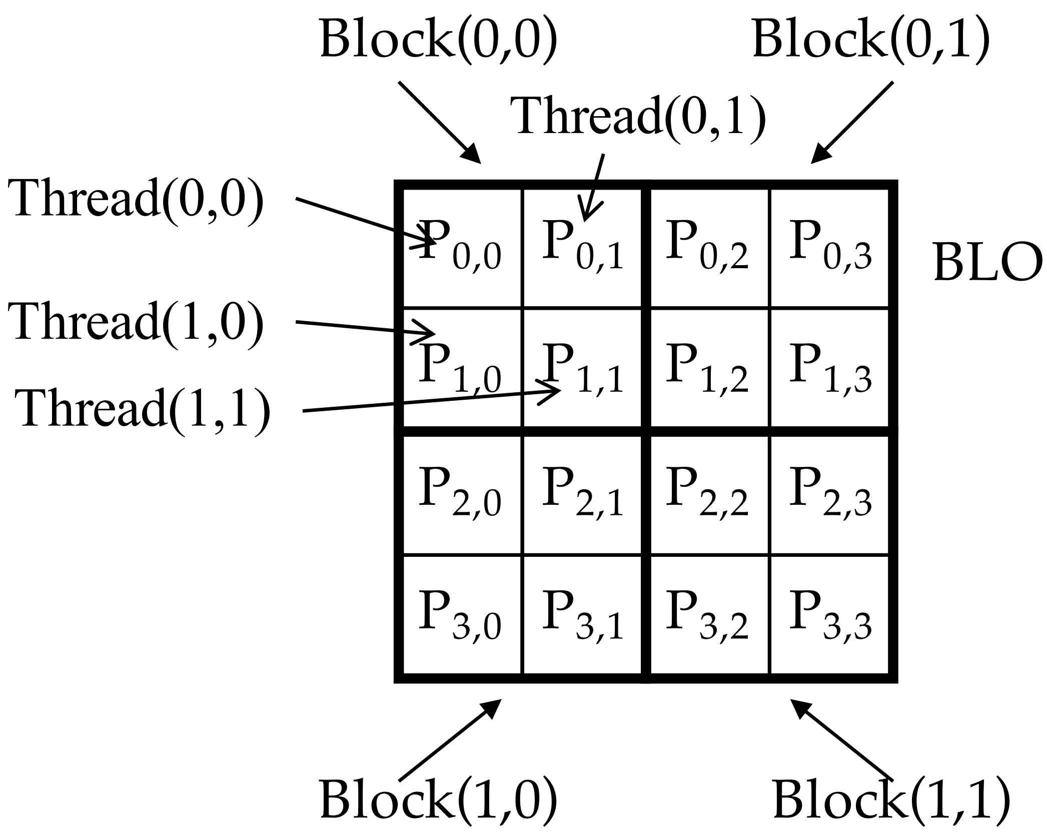
- When performing a matrix multiplication, each element of the output matrix P is an inner product of a row of M and a column of N.
- Similarly to previous examples each thread will compute a single value of the output matrix
- We organize threads as blocks



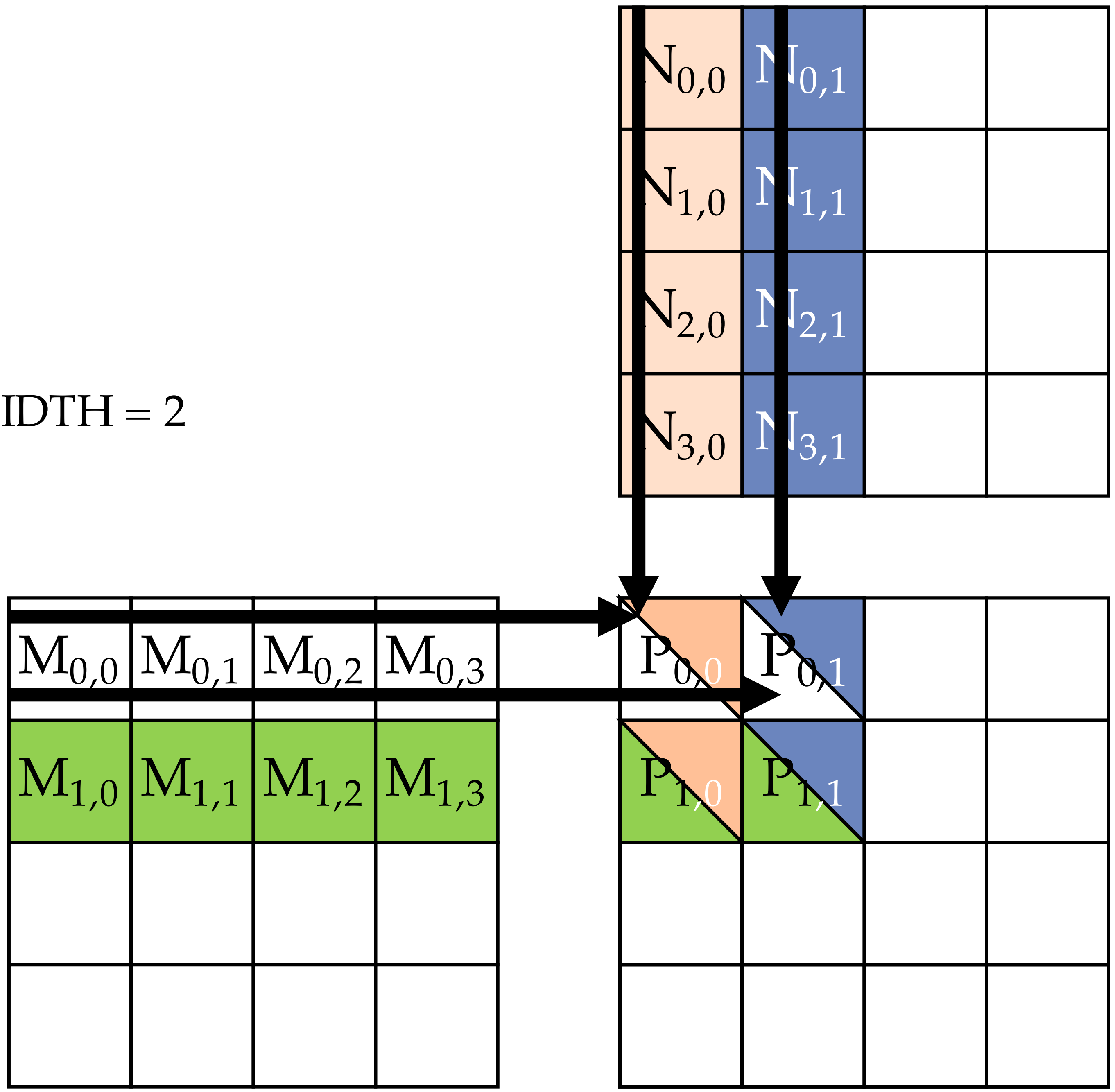
A solution

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width) {

  // Calculate the row index of the P element and M
  int Row = blockIdx.y*blockDim.y+threadIdx.y;

  // Calculate the column index of P and N
  int Col = blockIdx.x*blockDim.x+threadIdx.x;

  if ((Row < Width) && (Col < Width)) {
    float Pvalue = 0;
    // each thread computes one element of the block sub-matrix
    for (int k = 0; k < Width; ++k) {
      Pvalue += M[Row*Width+k]*N[k*Width+Col];
    }
    P[Row*Width+Col] = Pvalue;
  }

}
```

Toy example visualization


Block(0,0)
Block(0,1)
Thread(0,1)
Thread(0,0)
Thread(1,0)
Thread(1,1)
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$ $P_{0,3}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$ $P_{1,3}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$ $P_{2,3}$
$P_{3,0}$ $P_{3,1}$ $P_{3,2}$ $P_{3,3}$
Block(1,0)
Block(1,1)

BLOCK_WIDTH = 2

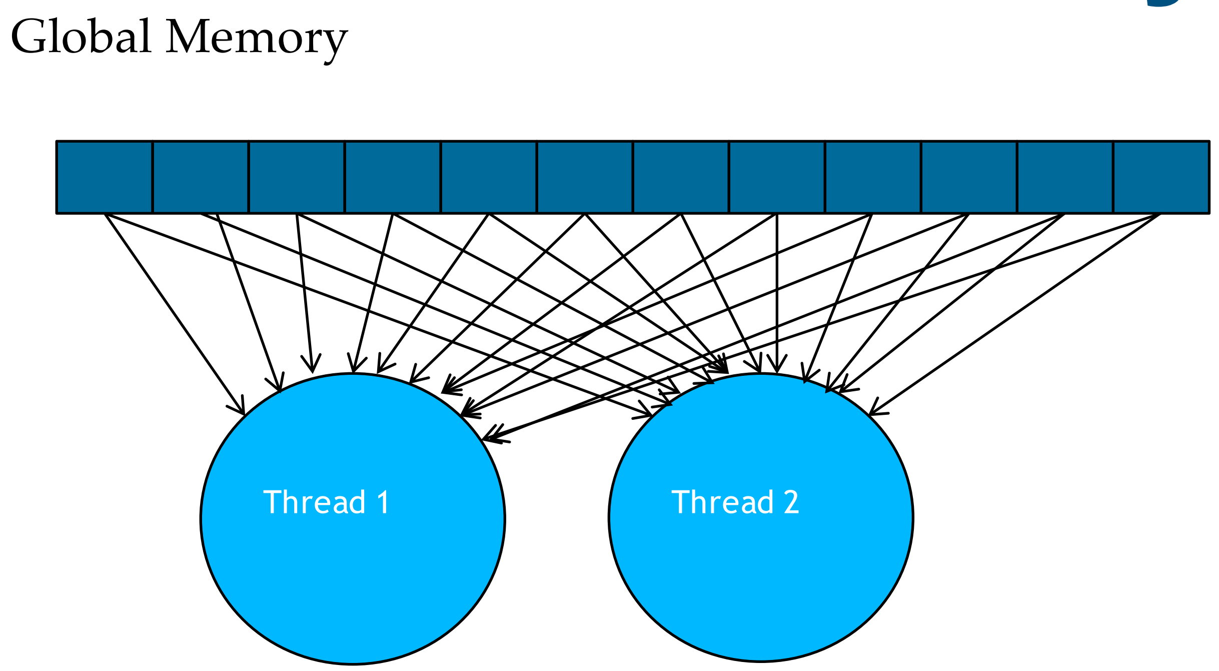


Reduce memory traffic: tiling

Tiling and CUDA memories

- Remind the tradeoffs of CUDA memories:
  - the global memory is large but slow;
  - the shared memory is small but fast.
- A common strategy is to partition the data into subsets called **tiles** so that each tile fits into the shared memory.
  An important criterion is that the kernel computation on these tiles can be done independently of each other.
  - Note that not all data structures can be partitioned into tiles given an arbitrary kernel function.

Global vs. shared memory access

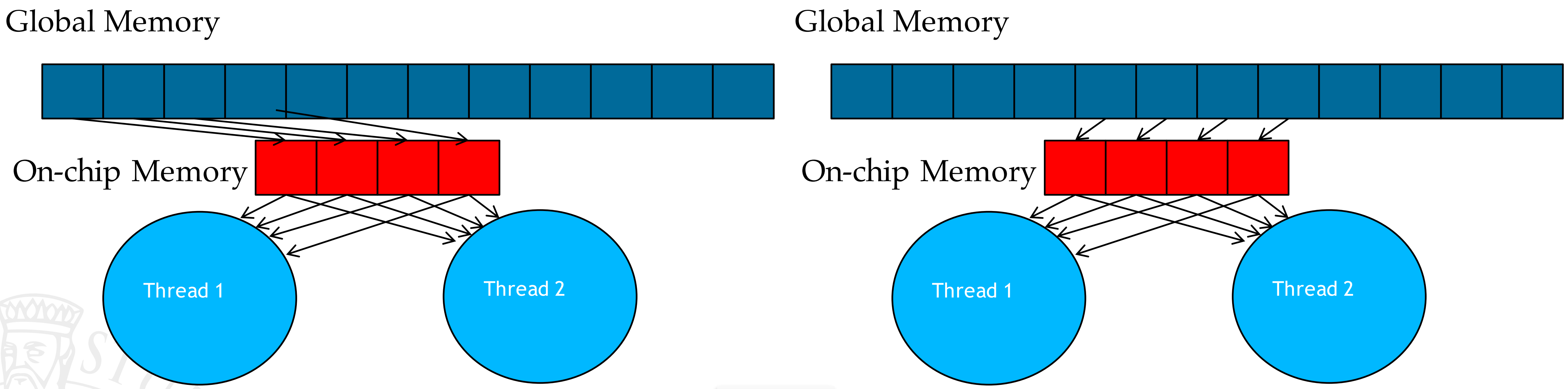


Global Memory

Thread 1

Thread 2

Bad



Global Memory

On-chip Memory

Thread 1

Thread 2

Global Memory

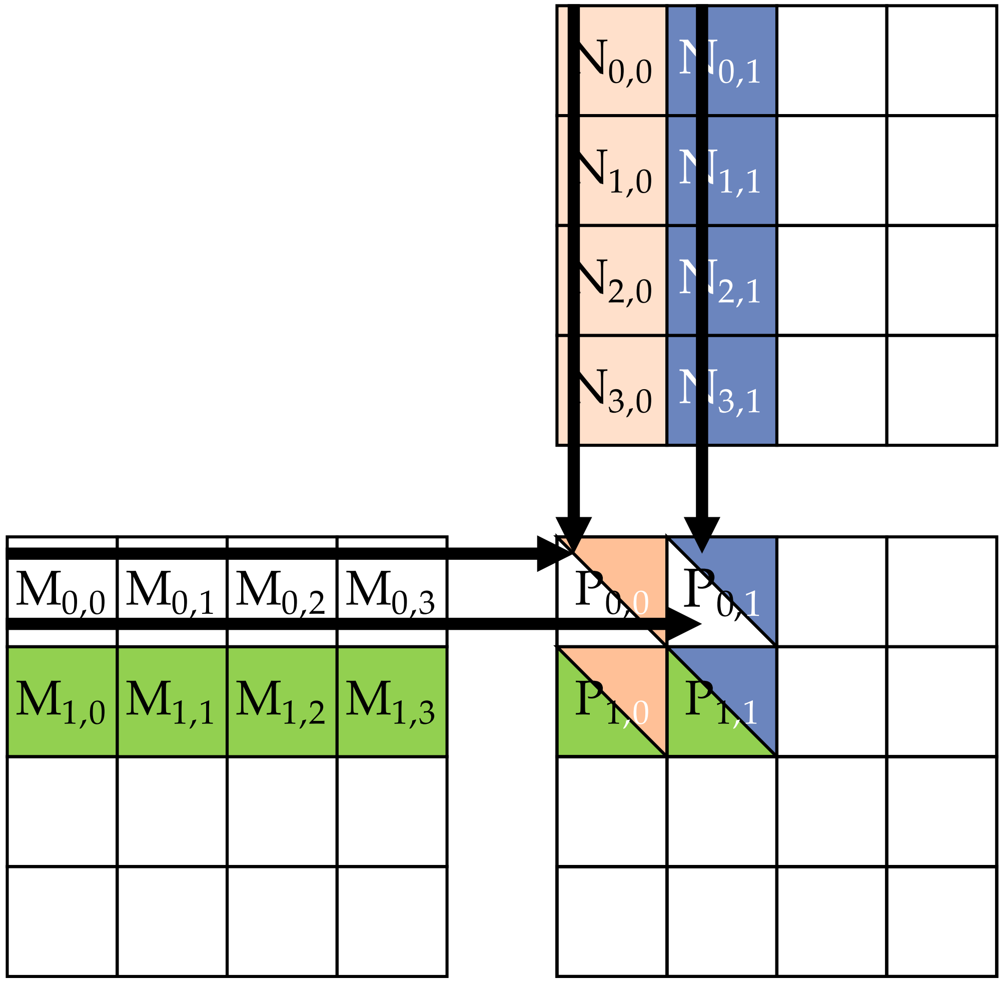
On-chip Memory

Thread 1

Thread 2

Good

Memory access in matrix multiplication



Access order →

$\text{thread}_{0,0}$	$\boxed{M_{0,0}} * N_{0,0}$	$M_{0,1} * \boxed{N_{1,0}}$	$M_{0,2} * N_{2,0}$	$M_{0,3} * N_{3,0}$
$\text{thread}_{0,1}$	$\boxed{M_{0,0}} * N_{0,1}$	$M_{0,1} * N_{1,1}$	$M_{0,2} * N_{2,1}$	$M_{0,3} * N_{3,1}$
$\text{thread}_{1,0}$	$M_{1,0} * N_{0,0}$	$M_{1,1} * \boxed{N_{1,0}}$	$M_{1,2} * N_{2,0}$	$M_{1,3} * N_{3,0}$
$\text{thread}_{1,1}$	$M_{1,0} * N_{0,1}$	$M_{1,1} * N_{1,1}$	$M_{1,2} * N_{2,1}$	$M_{1,3} * N_{3,1}$

- Each thread accesses four elements of M and four elements of N during its execution. Among the four threads highlighted, there is a significant overlap in the M and N elements they access. For example, $\text{thread}_{0,0}$ and $\text{thread}_{0,1}$ both access $M_{0,0}$ as well as the rest of row 0 of M.

- If we can somehow manage to have $\text{thread}_{0,0}$ and $\text{thread}_{0,1}$ to collaborate so that these M elements are only loaded from global memory once, we can reduce the total number of accesses to the global memory by half.

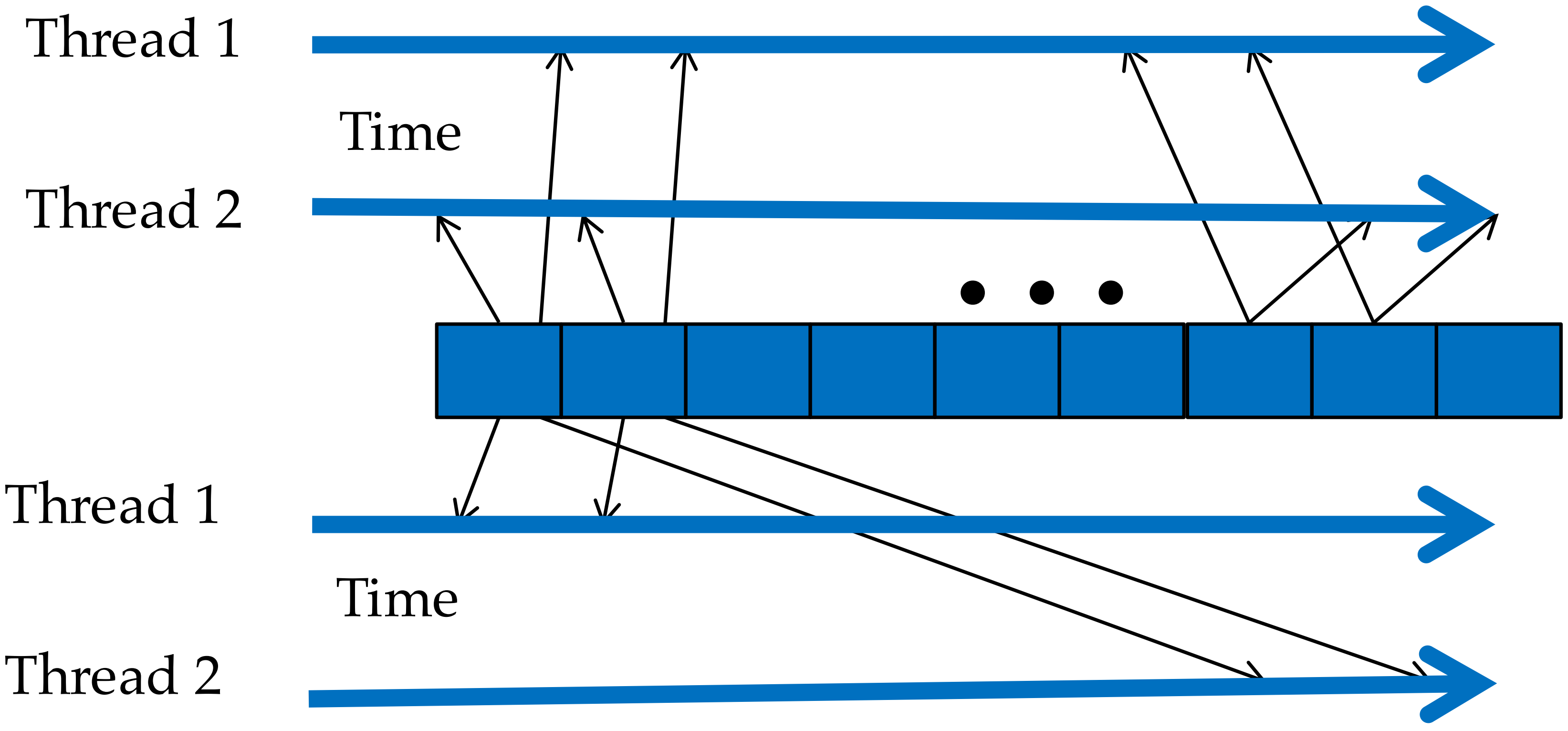
- In fact, we can see that every M and N element is accessed exactly twice during the execution of $\text{block}_{0,0}$.

- Larger block width leads to greater global memory traffic reduction.

Memory access patterns

Good: when threads have similar access timing

Bad: when threads have very different timing

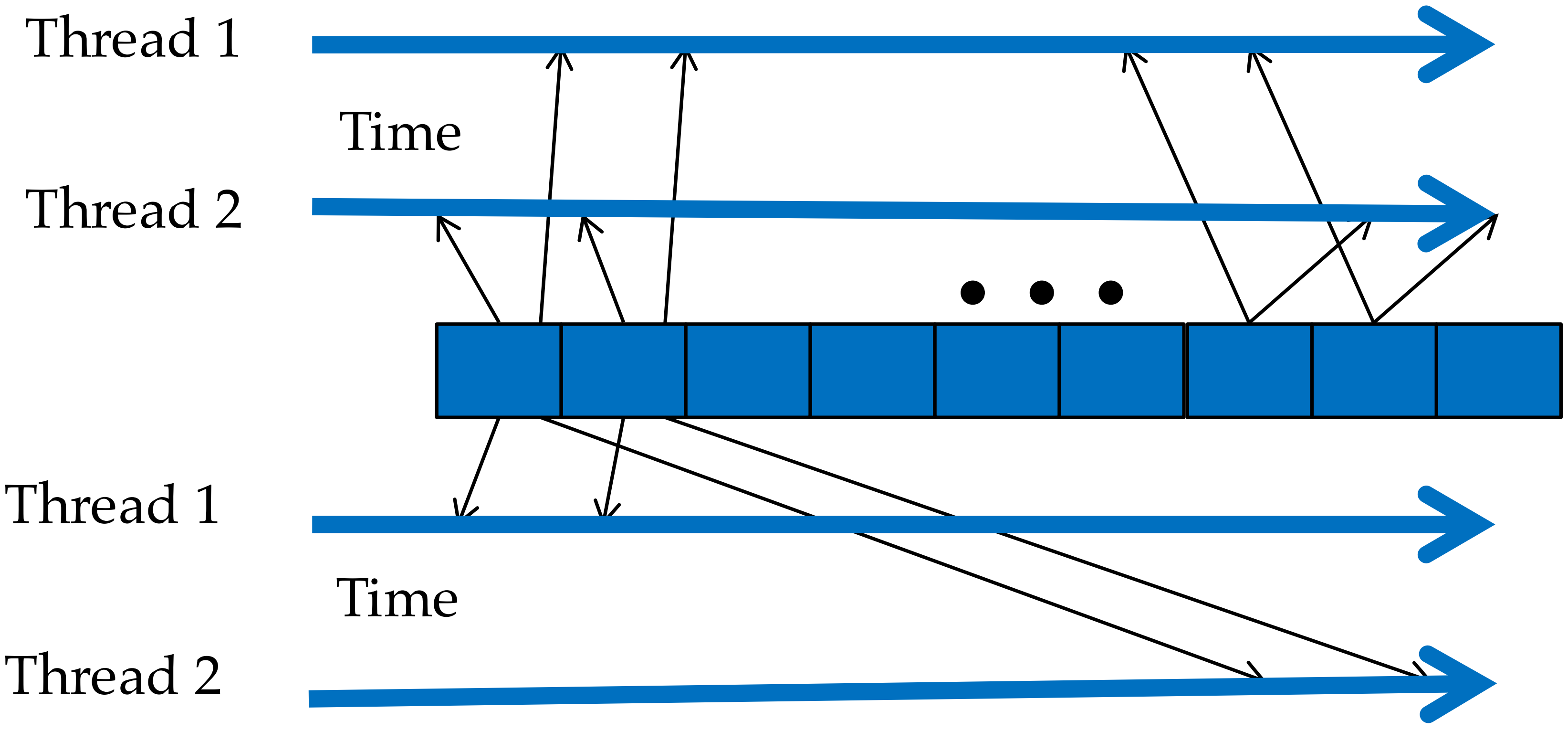


Memory access patterns

With tiling we are going to transform a program to localize memory locations accessed among threads and timing of their access.
Long access sequences of each thread are broken into phases, using barriers to synchronize access times to each section by the threads.

Good: when threads have similar access timing

Bad: when threads have very different timing

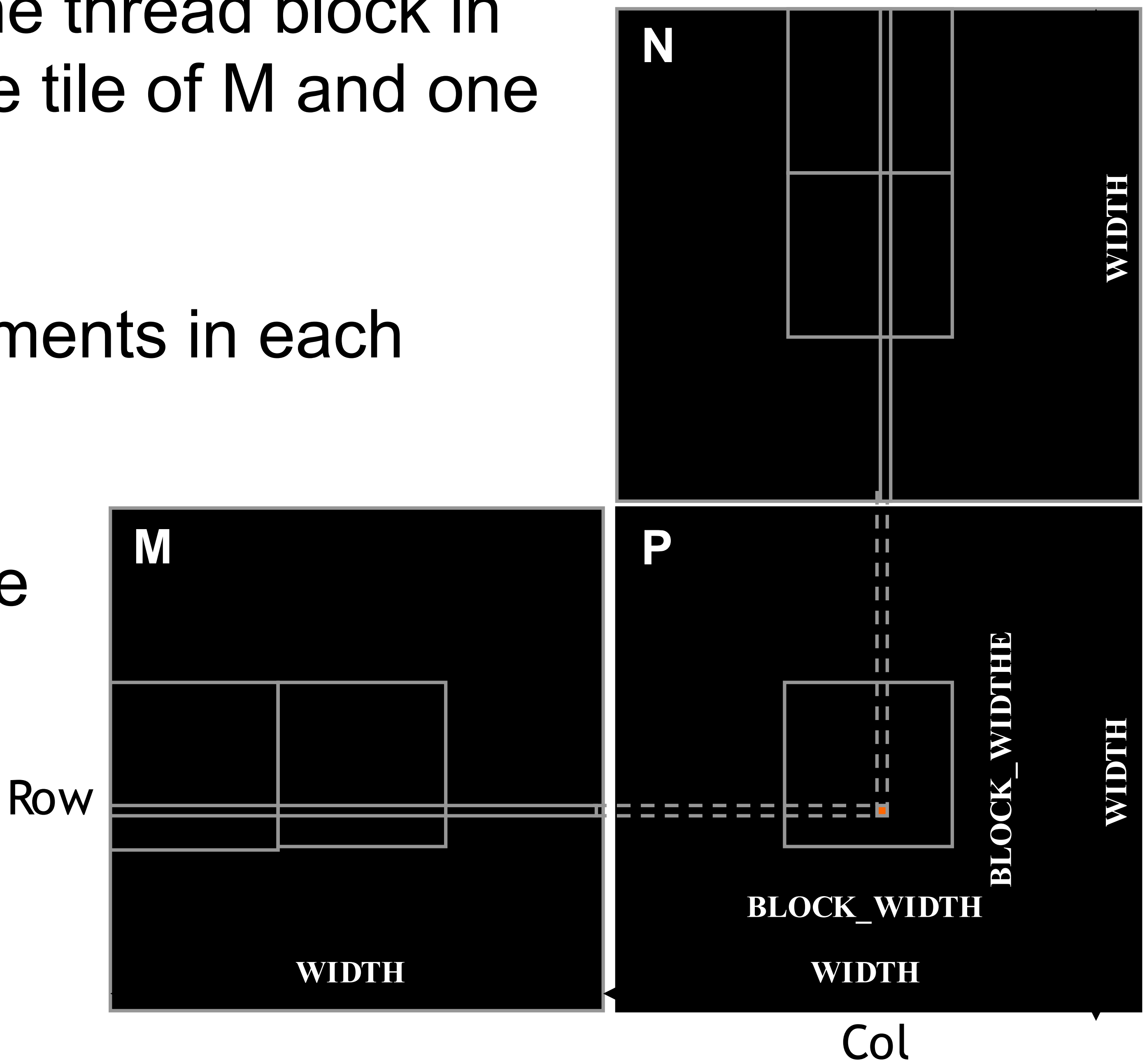


Tiling: how ?

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile

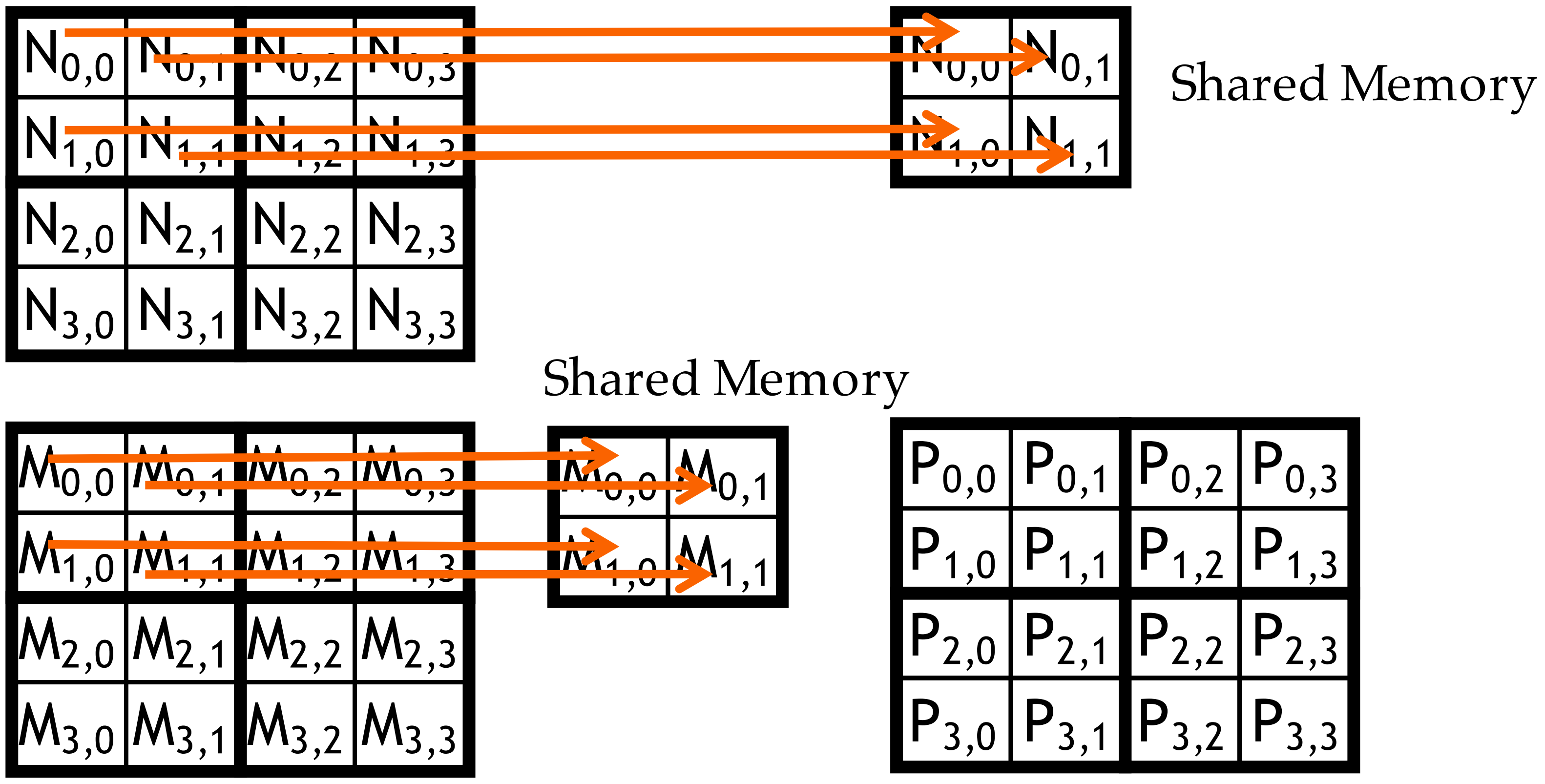
Tiling matrix multiplication

- Break up the execution of each thread into phases
- so that the data accesses by the thread block in each phase are focused on one tile of M and one tile of N
- The tile is of BLOCK_SIZE elements in each dimension
- All threads in a block participate
- Each thread loads one M element and one N element in tiled code



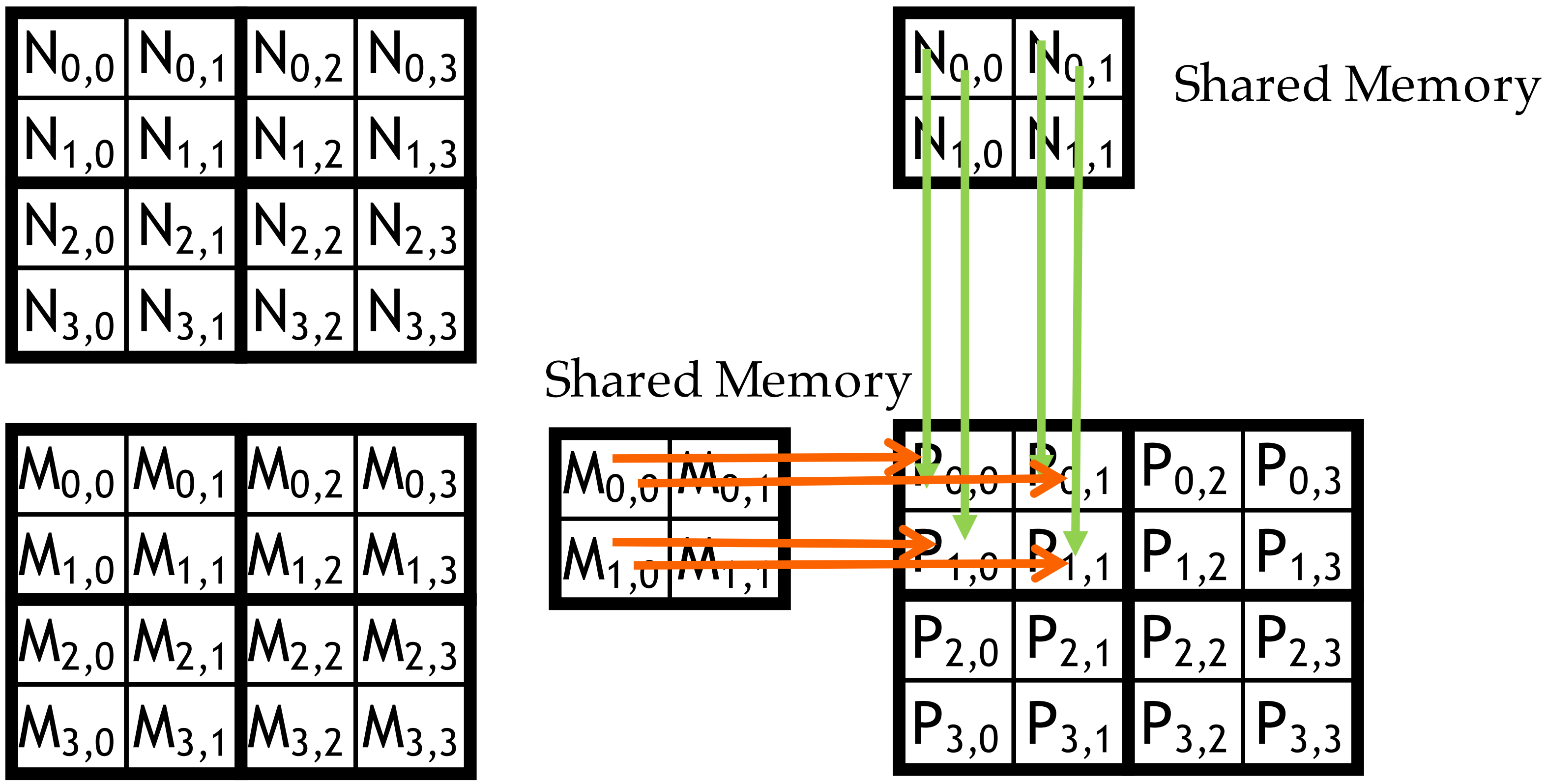
Tiling phases

Phase 0 Load for Block (0,0)


$N_{0,0}$ $N_{0,1}$ $N_{0,2}$ $N_{0,3}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$ $N_{1,3}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$ $N_{2,3}$
$N_{3,0}$ $N_{3,1}$ $N_{3,2}$ $N_{3,3}$
$N_{0,0}$ $N_{0,1}$
$N_{1,0}$ $N_{1,1}$
Shared Memory
Shared Memory
$M_{0,0}$ $M_{0,1}$ $M_{0,2}$ $M_{0,3}$
$M_{1,0}$ $M_{1,1}$ $M_{1,2}$ $M_{1,3}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$ $M_{2,3}$
$M_{3,0}$ $M_{3,1}$ $M_{3,2}$ $M_{3,3}$
$M_{0,0}$ $M_{0,1}$
$M_{1,0}$ $M_{1,1}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$ $P_{0,3}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$ $P_{1,3}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$ $P_{2,3}$
$P_{3,0}$ $P_{3,1}$ $P_{3,2}$ $P_{3,3}$

Tiling phases

Phase 0 Use for Block (0,0) (iteration 0)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$N_{0,0}$	$N_{0,1}$
$N_{1,0}$	$N_{1,1}$

Shared Memory

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

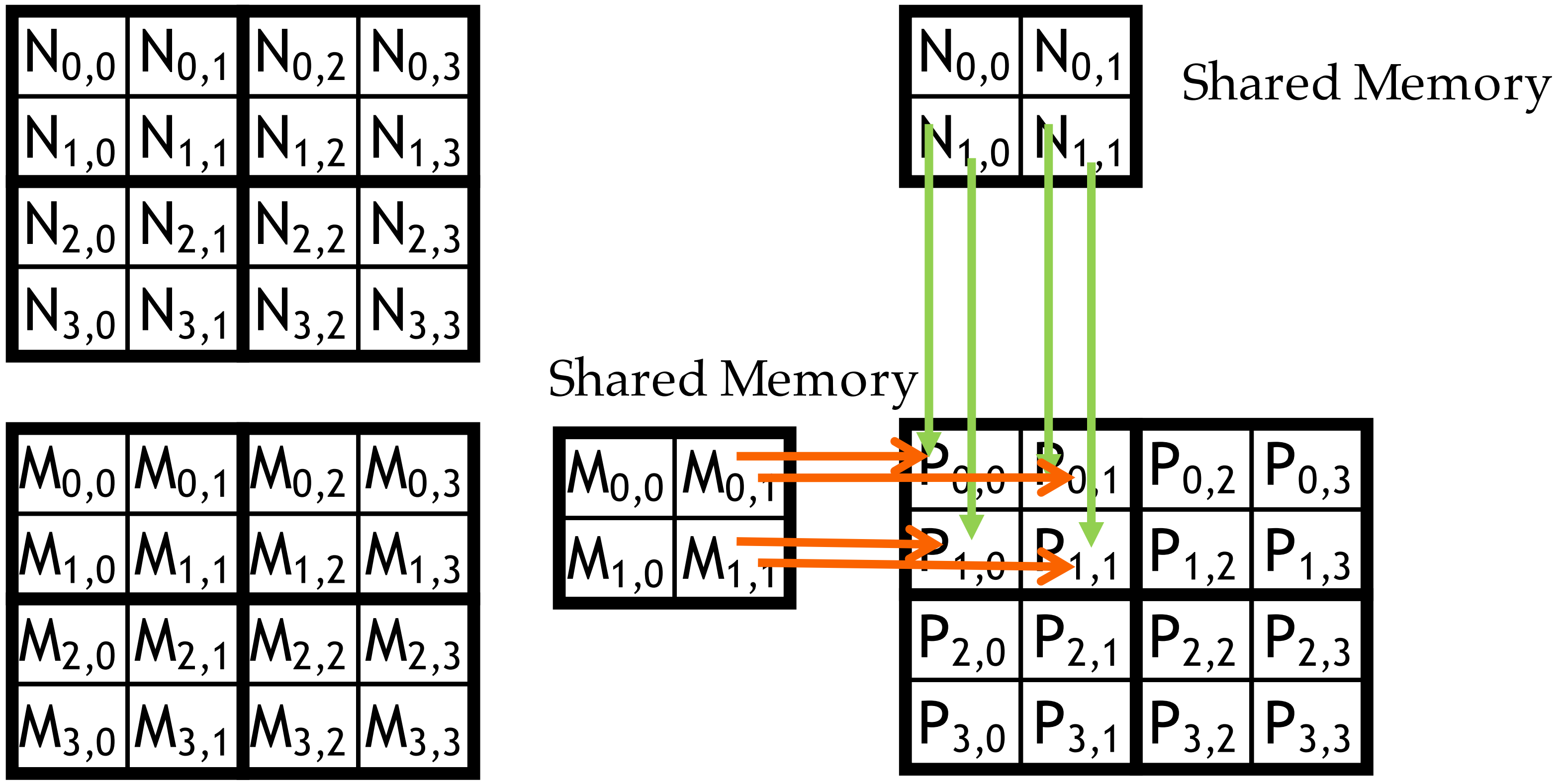
Shared Memory

$M_{0,0}$	$M_{0,1}$
$M_{1,0}$	$M_{1,1}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Tiling phases

Phase 0 Use for Block (0,0) (iteration 1)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

Shared Memory

$N_{0,0}$	$N_{0,1}$
$N_{1,0}$	$N_{1,1}$

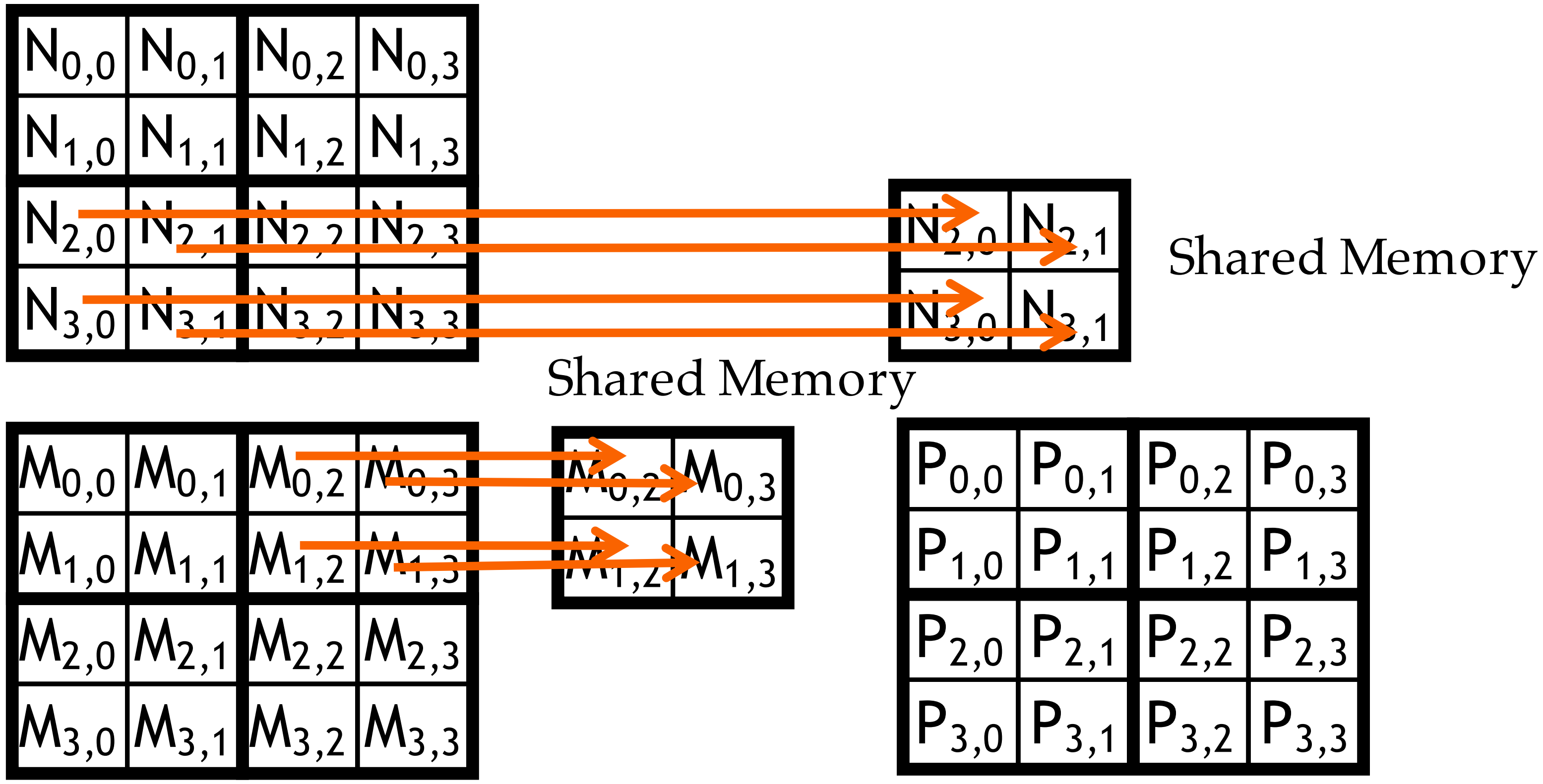
Shared Memory

$M_{0,0}$	$M_{0,1}$
$M_{1,0}$	$M_{1,1}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Tiling phases

Phase 1 Load for Block (0,0)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$N_{2,0}$	$N_{2,1}$
$N_{3,0}$	$N_{3,1}$

Shared Memory

Shared Memory

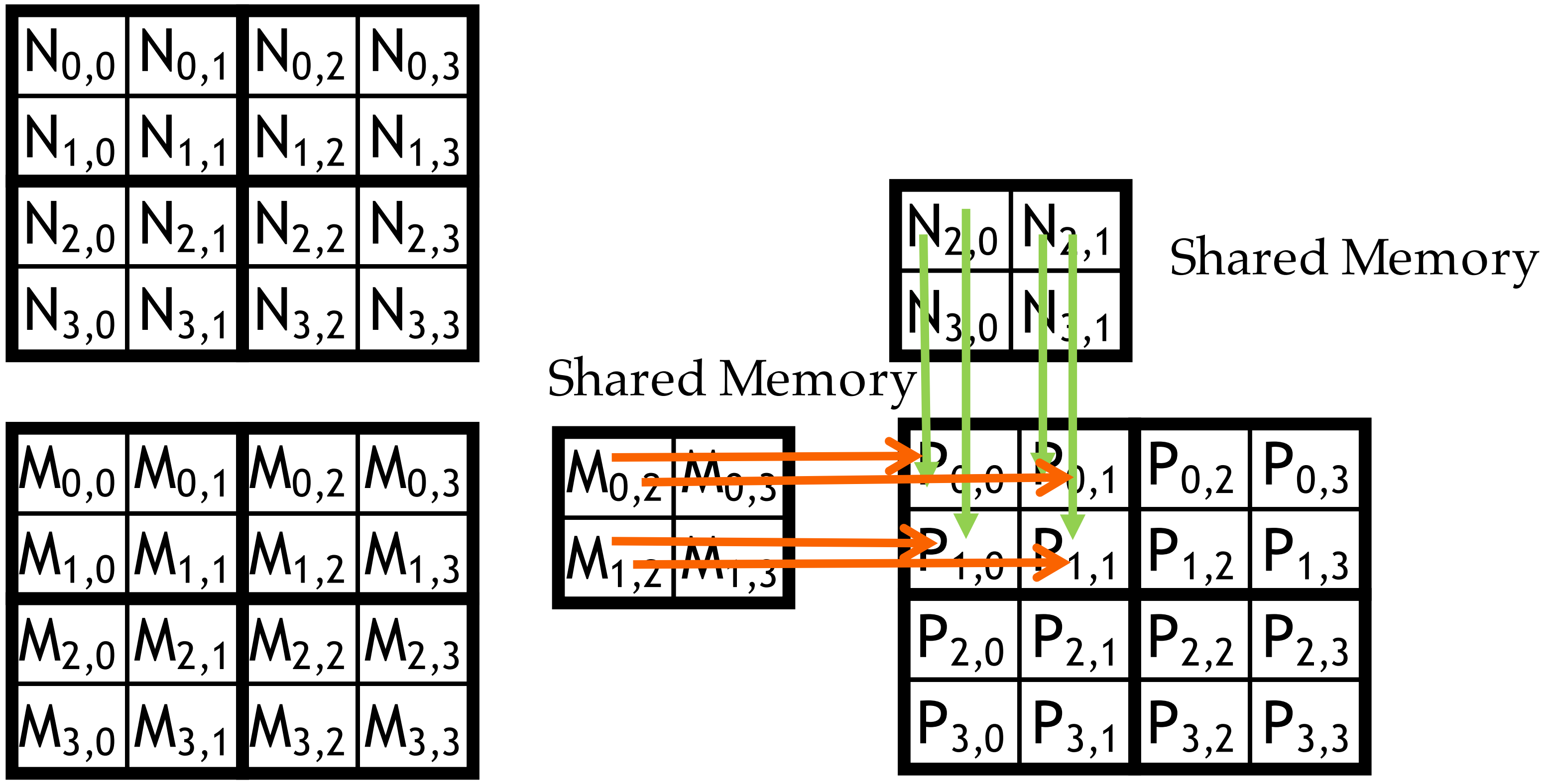
$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

$M_{0,2}$	$M_{0,3}$
$M_{1,2}$	$M_{1,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Tiling phases

Phase 1 Use for Block (0,0) (iteration 0)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

Shared Memory

$N_{2,0}$	$N_{2,1}$
$N_{3,0}$	$N_{3,1}$

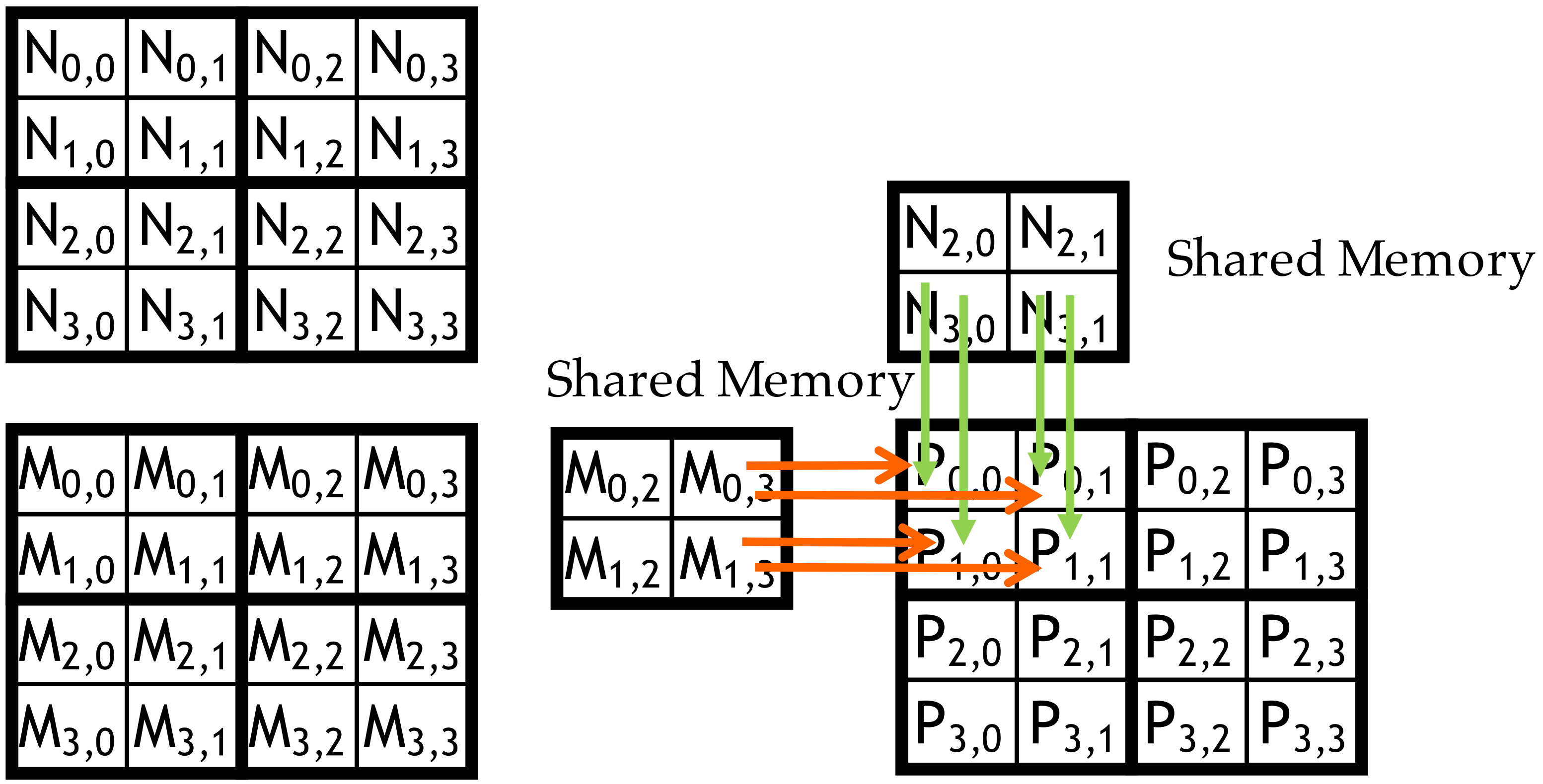
Shared Memory

$M_{0,2}$	$M_{0,3}$
$M_{1,2}$	$M_{1,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Tiling phases

Phase 1 Use for Block (0,0) (iteration 1)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	$N_{0,3}$
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$N_{1,3}$
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,0}$	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$

$N_{2,0}$	$N_{2,1}$
$N_{3,0}$	$N_{3,1}$

Shared Memory

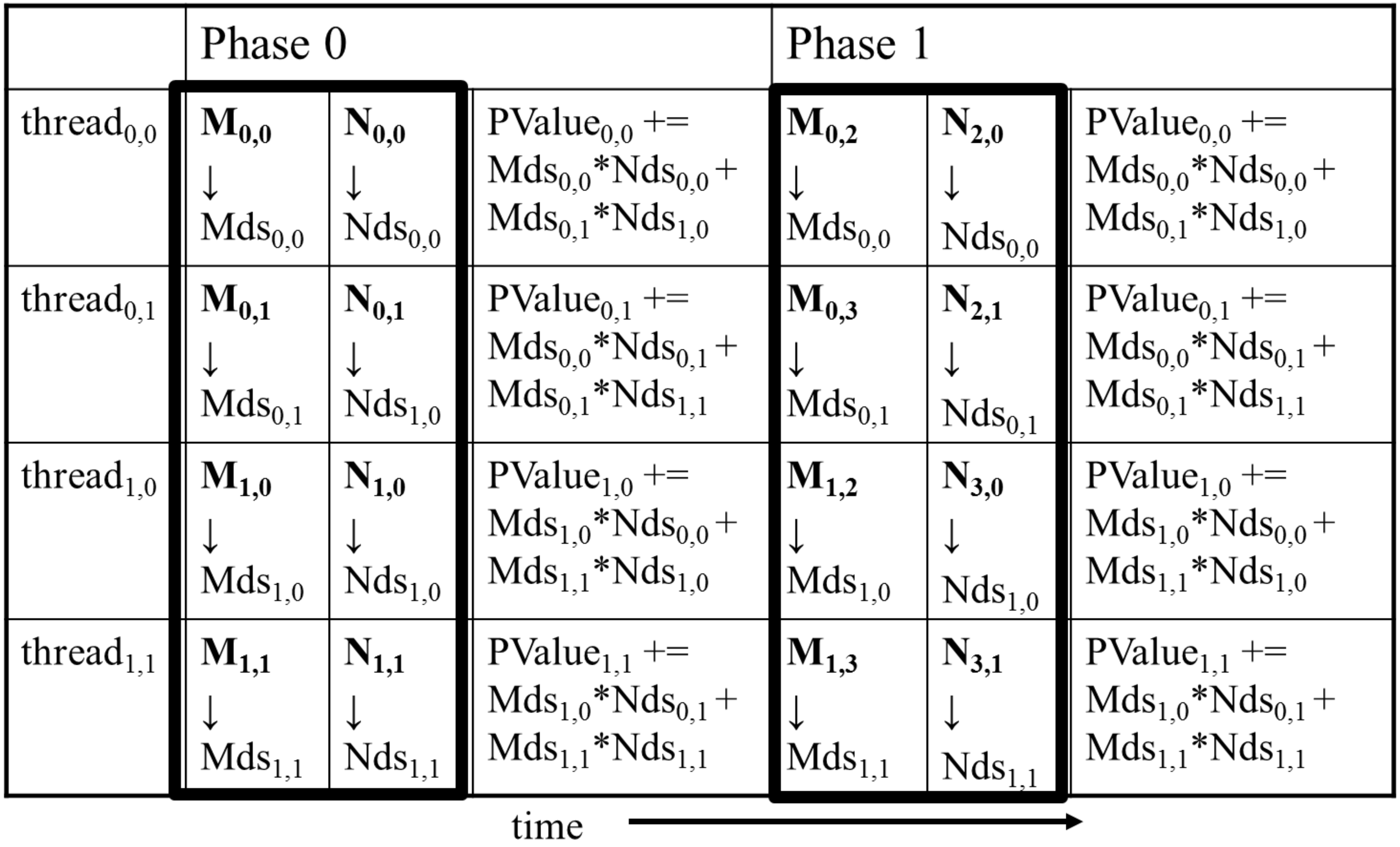
$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$

Shared Memory

$M_{0,2}$	$M_{0,3}$
$M_{1,2}$	$M_{1,3}$

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	$P_{0,3}$
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	$P_{2,3}$
$P_{3,0}$	$P_{3,1}$	$P_{3,2}$	$P_{3,3}$

Execution Phases of Toy Example

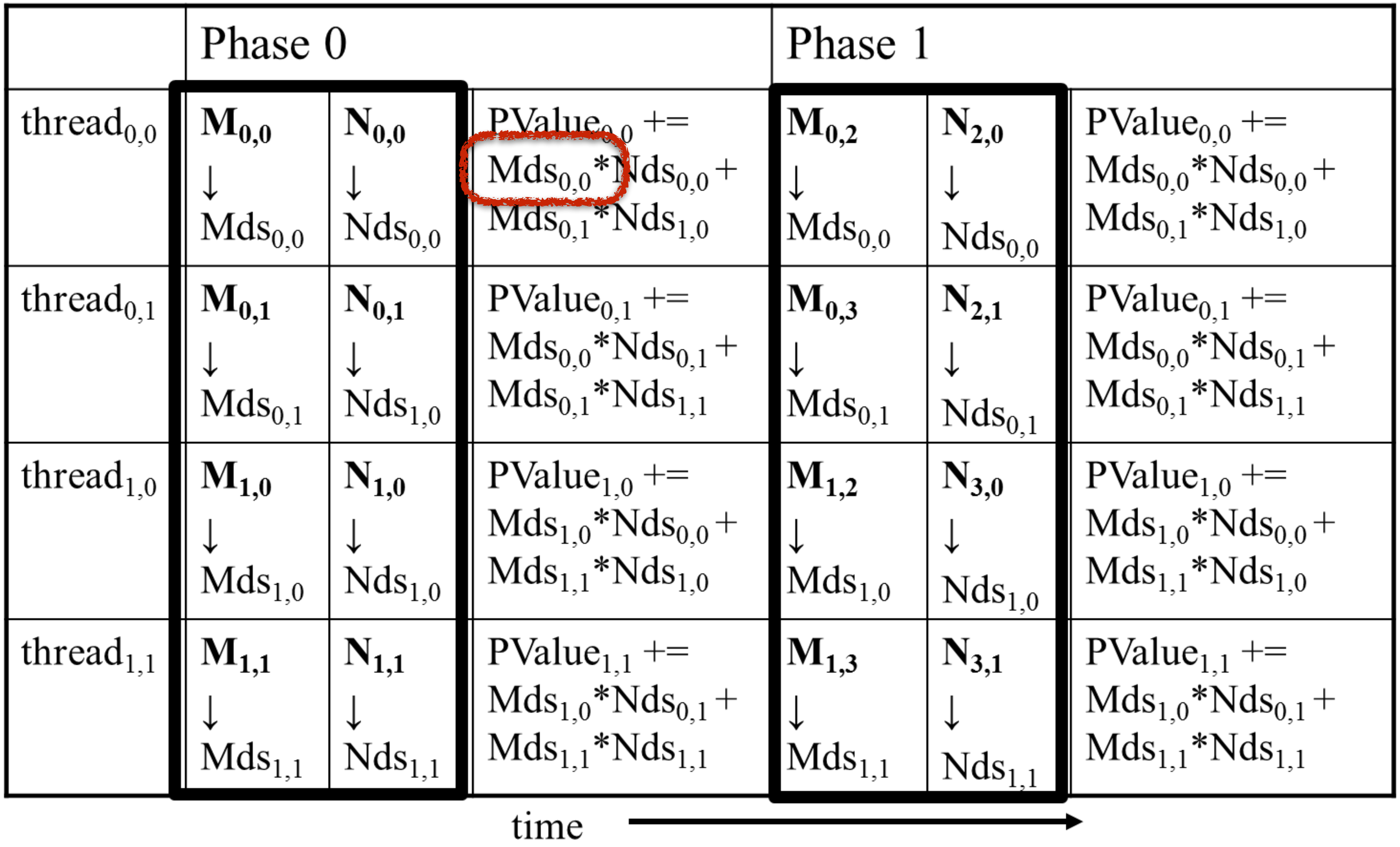


	Phase 0			Phase 1		
$\text{thread}_{0,0}$	$\mathbf{M}_{0,0}$ ↓ $\text{Mds}_{0,0}$	$\mathbf{N}_{0,0}$ ↓ $\text{Nds}_{0,0}$	$\text{PValue}_{0,0}$ += $\text{Mds}_{0,0}*\text{Nds}_{0,0}+$ $\text{Mds}_{0,1}*\text{Nds}_{1,0}$	$\mathbf{M}_{0,2}$ ↓ $\text{Mds}_{0,0}$	$\mathbf{N}_{2,0}$ ↓ $\text{Nds}_{0,0}$	$\text{PValue}_{0,0}$ += $\text{Mds}_{0,0}*\text{Nds}_{0,0}+$ $\text{Mds}_{0,1}*\text{Nds}_{1,0}$
$\text{thread}_{0,1}$	$\mathbf{M}_{0,1}$ ↓ $\text{Mds}_{0,1}$	$\mathbf{N}_{0,1}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{0,1}$ += $\text{Mds}_{0,0}*\text{Nds}_{0,1}+$ $\text{Mds}_{0,1}*\text{Nds}_{1,1}$	$\mathbf{M}_{0,3}$ ↓ $\text{Mds}_{0,1}$	$\mathbf{N}_{2,1}$ ↓ $\text{Nds}_{0,1}$	$\text{PValue}_{0,1}$ += $\text{Mds}_{0,0}*\text{Nds}_{0,1}+$ $\text{Mds}_{0,1}*\text{Nds}_{1,1}$
$\text{thread}_{1,0}$	$\mathbf{M}_{1,0}$ ↓ $\text{Mds}_{1,0}$	$\mathbf{N}_{1,0}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{1,0}$ += $\text{Mds}_{1,0}*\text{Nds}_{0,0}+$ $\text{Mds}_{1,1}*\text{Nds}_{1,0}$	$\mathbf{M}_{1,2}$ ↓ $\text{Mds}_{1,0}$	$\mathbf{N}_{3,0}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{1,0}$ += $\text{Mds}_{1,0}*\text{Nds}_{0,0}+$ $\text{Mds}_{1,1}*\text{Nds}_{1,0}$
$\text{thread}_{1,1}$	$\mathbf{M}_{1,1}$ ↓ $\text{Mds}_{1,1}$	$\mathbf{N}_{1,1}$ ↓ $\text{Nds}_{1,1}$	$\text{PValue}_{1,1}$ += $\text{Mds}_{1,0}*\text{Nds}_{0,1}+$ $\text{Mds}_{1,1}*\text{Nds}_{1,1}$	$\mathbf{M}_{1,3}$ ↓ $\text{Mds}_{1,1}$	$\mathbf{N}_{3,1}$ ↓ $\text{Nds}_{1,1}$	$\text{PValue}_{1,1}$ += $\text{Mds}_{1,0}*\text{Nds}_{0,1}+$ $\text{Mds}_{1,1}*\text{Nds}_{1,1}$

time

- Shared memory allows each value to be accessed by multiple threads

Execution Phases of Toy Example

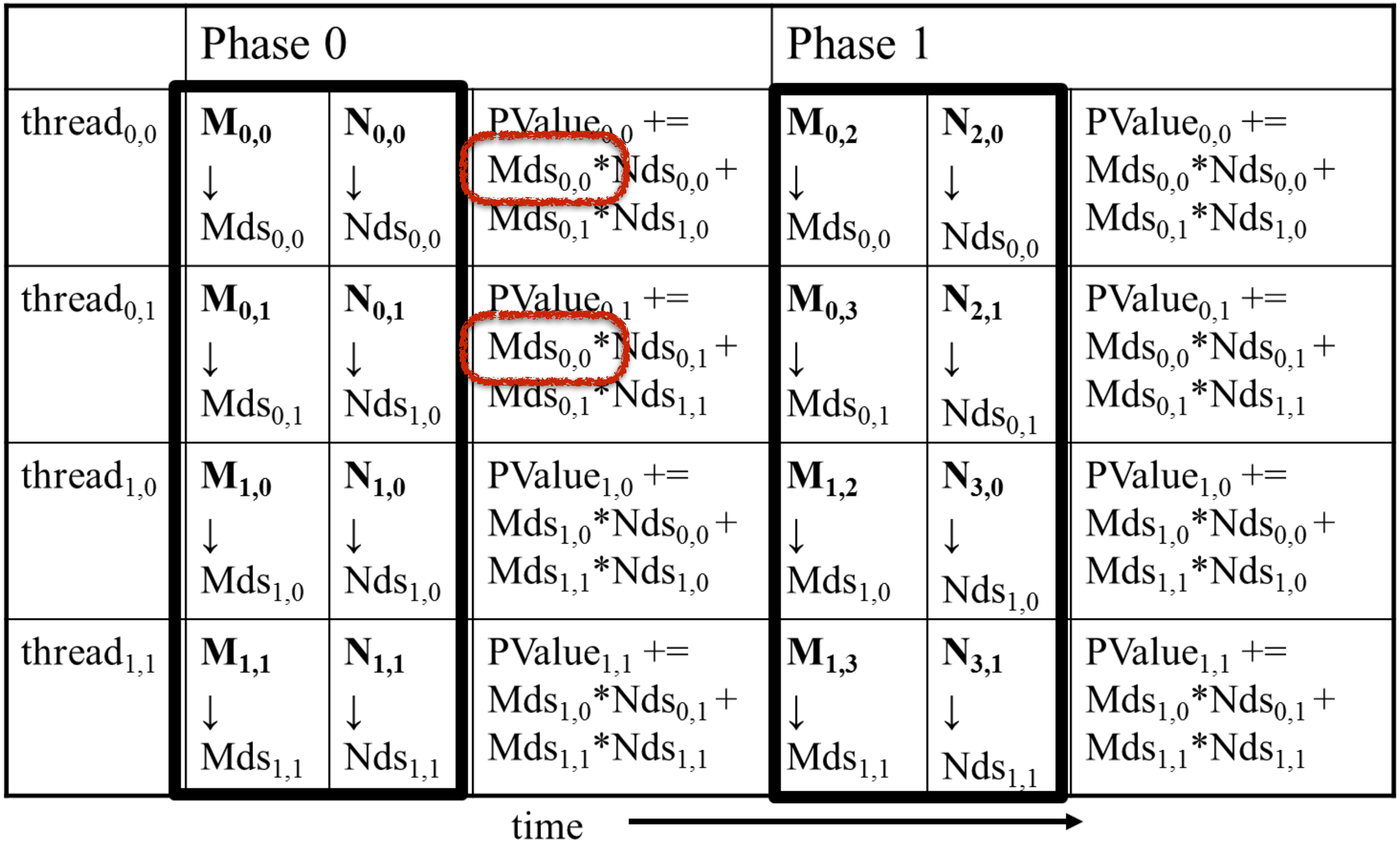


	Phase 0			Phase 1		
$\text{thread}_{0,0}$	$\mathbf{M}_{0,0}$ ↓ $\text{Mds}_{0,0}$	$\mathbf{N}_{0,0}$ ↓ $\text{Nds}_{0,0}$	$\text{PValue}_{0,0}$ += $\text{Mds}_{0,0}$*$\text{Nds}_{0,0}$ + $\text{Mds}_{0,1}$*$\text{Nds}_{1,0}$	$\mathbf{M}_{0,2}$ ↓ $\text{Mds}_{0,0}$	$\mathbf{N}_{2,0}$ ↓ $\text{Nds}_{0,0}$	$\text{PValue}_{0,0}$ += $\text{Mds}_{0,0}$*$\text{Nds}_{0,0}$ + $\text{Mds}_{0,1}$*$\text{Nds}_{1,0}$
$\text{thread}_{0,1}$	$\mathbf{M}_{0,1}$ ↓ $\text{Mds}_{0,1}$	$\mathbf{N}_{0,1}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{0,1}$ += $\text{Mds}_{0,0}$*$\text{Nds}_{0,1}$ + $\text{Mds}_{0,1}$*$\text{Nds}_{1,1}$	$\mathbf{M}_{0,3}$ ↓ $\text{Mds}_{0,1}$	$\mathbf{N}_{2,1}$ ↓ $\text{Nds}_{0,1}$	$\text{PValue}_{0,1}$ += $\text{Mds}_{0,0}$*$\text{Nds}_{0,1}$ + $\text{Mds}_{0,1}$*$\text{Nds}_{1,1}$
$\text{thread}_{1,0}$	$\mathbf{M}_{1,0}$ ↓ $\text{Mds}_{1,0}$	$\mathbf{N}_{1,0}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{1,0}$ += $\text{Mds}_{1,0}$*$\text{Nds}_{0,0}$ + $\text{Mds}_{1,1}$*$\text{Nds}_{1,0}$	$\mathbf{M}_{1,2}$ ↓ $\text{Mds}_{1,0}$	$\mathbf{N}_{3,0}$ ↓ $\text{Nds}_{1,0}$	$\text{PValue}_{1,0}$ += $\text{Mds}_{1,0}$*$\text{Nds}_{0,0}$ + $\text{Mds}_{1,1}$*$\text{Nds}_{1,0}$
$\text{thread}_{1,1}$	$\mathbf{M}_{1,1}$ ↓ $\text{Mds}_{1,1}$	$\mathbf{N}_{1,1}$ ↓ $\text{Nds}_{1,1}$	$\text{PValue}_{1,1}$ += $\text{Mds}_{1,0}$*$\text{Nds}_{0,1}$ + $\text{Mds}_{1,1}$*$\text{Nds}_{1,1}$	$\mathbf{M}_{1,3}$ ↓ $\text{Mds}_{1,1}$	$\mathbf{N}_{3,1}$ ↓ $\text{Nds}_{1,1}$	$\text{PValue}_{1,1}$ += $\text{Mds}_{1,0}$*$\text{Nds}_{0,1}$ + $\text{Mds}_{1,1}$*$\text{Nds}_{1,1}$

time →

- Shared memory allows each value to be accessed by multiple threads

Execution Phases of Toy Example

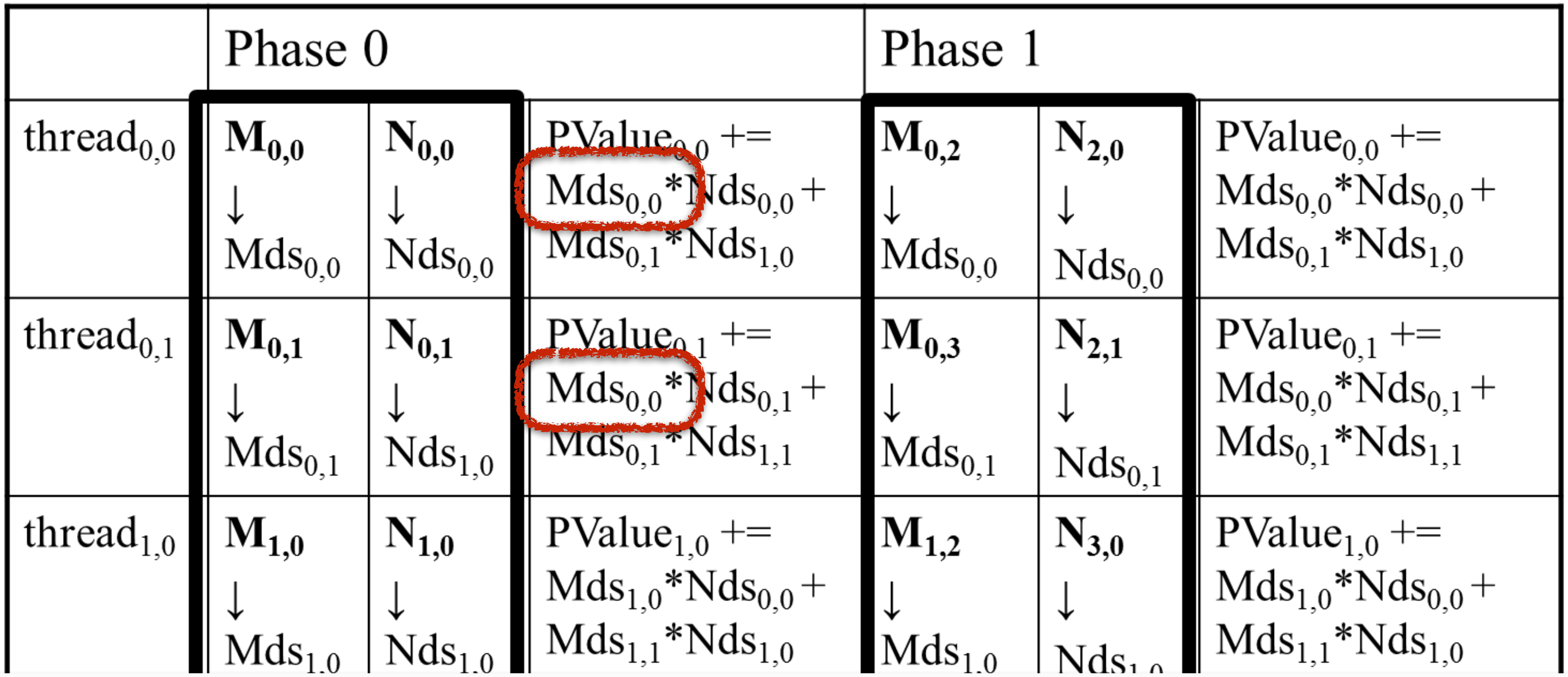


	Phase 0			Phase 1		
$thread_{0,0}$	$\mathbf{M_{0,0}}$ ↓ $Mds_{0,0}$	$\mathbf{N_{0,0}}$ ↓ $Nds_{0,0}$	$PValue_{0,0}$ += $Mds_{0,0}*Nds_{0,0}$ + $Mds_{0,1}*Nds_{1,0}$	$\mathbf{M_{0,2}}$ ↓ $Mds_{0,0}$	$\mathbf{N_{2,0}}$ ↓ $Nds_{0,0}$	$PValue_{0,0}$ += $Mds_{0,0}*Nds_{0,0}$ + $Mds_{0,1}*Nds_{1,0}$
$thread_{0,1}$	$\mathbf{M_{0,1}}$ ↓ $Mds_{0,1}$	$\mathbf{N_{0,1}}$ ↓ $Nds_{1,0}$	$PValue_{0,1}$ += $Mds_{0,0}*Nds_{0,1}$ + $Mds_{0,1}*Nds_{1,1}$	$\mathbf{M_{0,3}}$ ↓ $Mds_{0,1}$	$\mathbf{N_{2,1}}$ ↓ $Nds_{0,1}$	$PValue_{0,1}$ += $Mds_{0,0}*Nds_{0,1}$ + $Mds_{0,1}*Nds_{1,1}$
$thread_{1,0}$	$\mathbf{M_{1,0}}$ ↓ $Mds_{1,0}$	$\mathbf{N_{1,0}}$ ↓ $Nds_{1,0}$	$PValue_{1,0}$ += $Mds_{1,0}*Nds_{0,0}$ + $Mds_{1,1}*Nds_{1,0}$	$\mathbf{M_{1,2}}$ ↓ $Mds_{1,0}$	$\mathbf{N_{3,0}}$ ↓ $Nds_{1,0}$	$PValue_{1,0}$ += $Mds_{1,0}*Nds_{0,0}$ + $Mds_{1,1}*Nds_{1,0}$
$thread_{1,1}$	$\mathbf{M_{1,1}}$ ↓ $Mds_{1,1}$	$\mathbf{N_{1,1}}$ ↓ $Nds_{1,1}$	$PValue_{1,1}$ += $Mds_{1,0}*Nds_{0,1}$ + $Mds_{1,1}*Nds_{1,1}$	$\mathbf{M_{1,3}}$ ↓ $Mds_{1,1}$	$\mathbf{N_{3,1}}$ ↓ $Nds_{1,1}$	$PValue_{1,1}$ += $Mds_{1,0}*Nds_{0,1}$ + $Mds_{1,1}*Nds_{1,1}$

time →

- Shared memory allows each value to be accessed by multiple threads

Execution Phases of Toy Example



	Phase 0			Phase 1		
thread$_{0,0}$	$\mathbf{M}_{0,0}$ ↓ Mds$_{0,0}$	$\mathbf{N}_{0,0}$ ↓ Nds$_{0,0}$	PValue$_{0,0}$ += Mds$_{0,0}$*Nds$_{0,0}$ + Mds$_{0,1}$*Nds$_{1,0}$	$\mathbf{M}_{0,2}$ ↓ Mds$_{0,0}$	$\mathbf{N}_{2,0}$ ↓ Nds$_{0,0}$	PValue$_{0,0}$ += Mds$_{0,0}$*Nds$_{0,0}$ + Mds$_{0,1}$*Nds$_{1,0}$
thread$_{0,1}$	$\mathbf{M}_{0,1}$ ↓ Mds$_{0,1}$	$\mathbf{N}_{0,1}$ ↓ Nds$_{1,0}$	PValue$_{0,1}$ += Mds$_{0,0}$*Nds$_{0,1}$ + Mds$_{0,1}$*Nds$_{1,1}$	$\mathbf{M}_{0,3}$ ↓ Mds$_{0,1}$	$\mathbf{N}_{2,1}$ ↓ Nds$_{0,1}$	PValue$_{0,1}$ += Mds$_{0,0}$*Nds$_{0,1}$ + Mds$_{0,1}$*Nds$_{1,1}$
thread$_{1,0}$	$\mathbf{M}_{1,0}$ ↓ Mds$_{1,0}$	$\mathbf{N}_{1,0}$ ↓ Nds$_{1,0}$	PValue$_{1,0}$ += Mds$_{1,0}$*Nds$_{0,0}$ + Mds$_{1,1}$*Nds$_{1,0}$	$\mathbf{M}_{1,2}$ ↓ Mds$_{1,0}$	$\mathbf{N}_{3,0}$ ↓ Nds$_{1,0}$	PValue$_{1,0}$ += Mds$_{1,0}$*Nds$_{0,0}$ + Mds$_{1,1}$*Nds$_{1,0}$

- In general, if an input matrix is of dimension Width and the tile size is TILE_WIDTH, the dot product would be performed in Width/TILE_WIDTH phases.
- The creation of these phases is key to the reduction of accesses to the global memory.
- With each phase focusing on a small subset of the input matrix values, the threads can collaboratively load the subset into the shared memory and use the values in the shared memory to satisfy their overlapping input needs in the phase.
- Mds and Nds are re-used to hold the input values in different phases: reducing need of shared memory.

Barrier Synchronization

- Synchronize all threads in a block
  - `__syncthreads()`
- All threads in the same block must reach the `__syncthreads()` before any of the them can move on
- Best used to coordinate the phased execution tiled algorithms
  - To ensure that all elements of a tile are loaded at the beginning of a phase
  - To ensure that all elements of a tile are consumed at the end of a phase

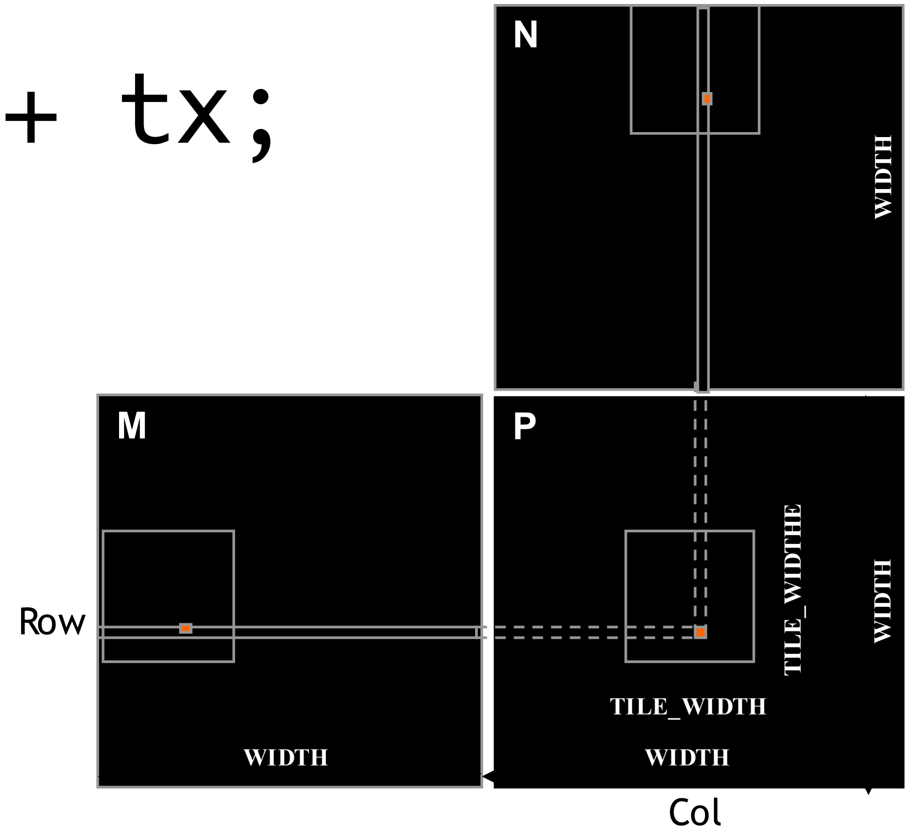
Loading Input Tile 0 of M (Phase 0)

- Have each thread load an M element and an N element at the same relative position as its P element.

- `int Row = by * blockDim.y + ty;`

- `int Col = bx * blockDim.x + tx;`

- 2D indexing for accessing Tile 0:
  M[Row][tx]
  N[ty][Col]



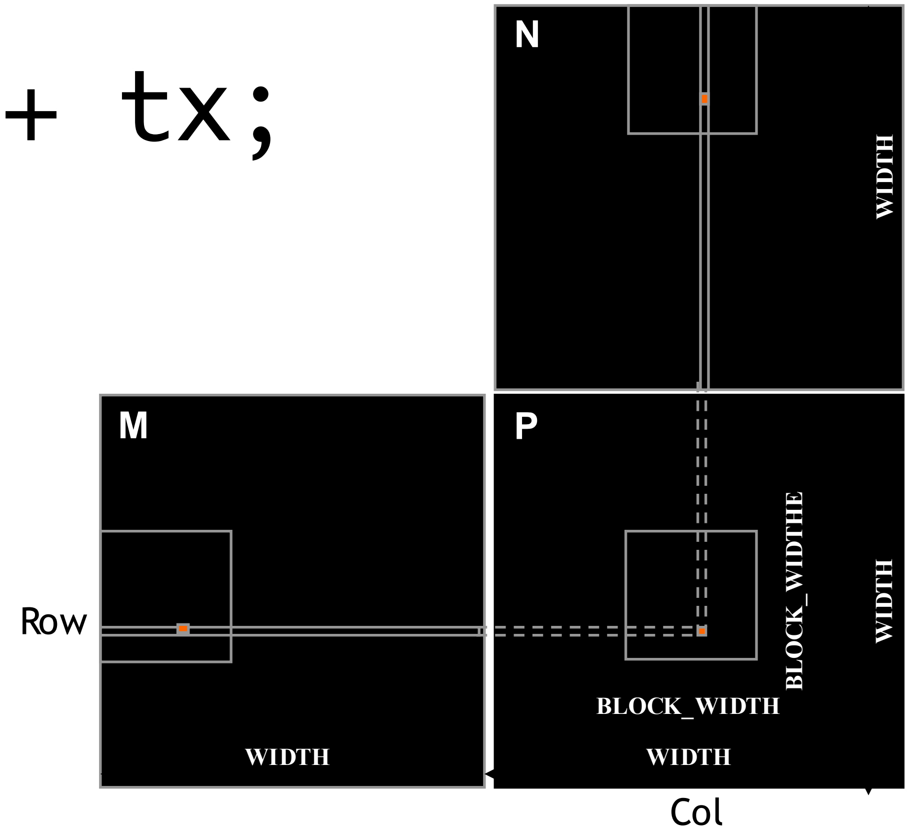
Loading Input Tile 0 of N (Phase 0)

- Have each thread load an M element and an N element at the same relative position as its P element.

- `int Row = by * blockDim.y + ty;`

- `int Col = bx * blockDim.x + tx;`

- 2D indexing for accessing Tile 0:
  M[Row][tx]
  N[ty][Col]

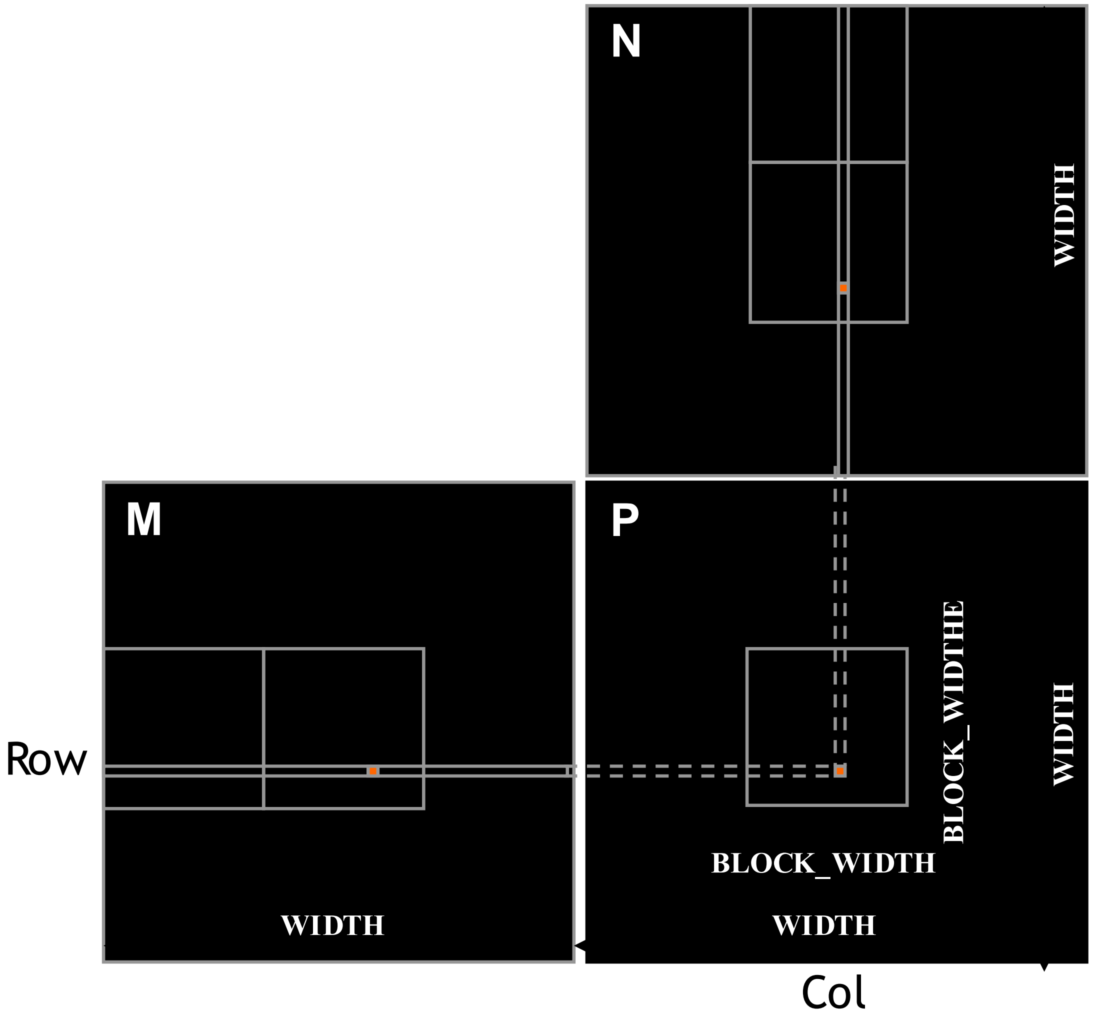


Loading Input Tile 1 of M (Phase 1)

- 2D indexing for accessing Tile 1:

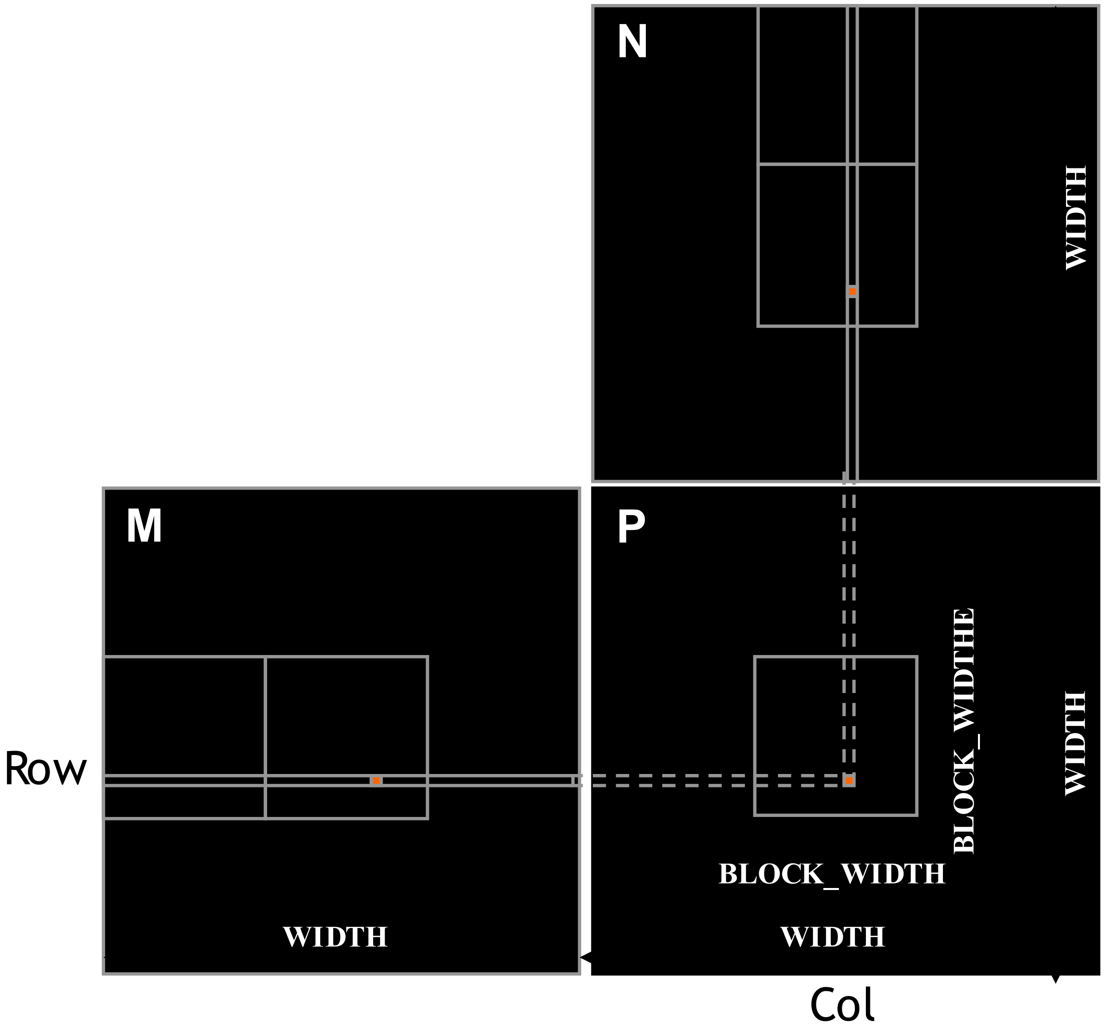
  M[Row][1*TILE_WIDTH + tx]

  N[1*TILE_WIDTH + ty][Col]



Loading Input Tile 1 of N (Phase 1)

- 2D indexing for accessing Tile 1:
  M[Row][1*TILE_WIDTH + tx]
  N[1*TILE_WIDTH + ty][Col]



Allocating M and N

- M and N can be allocated dynamically, using 1D indexing

- `M[Row][ph*TILE_WIDTH+tx]`

- `M[Row*Width + ph*TILE_WIDTH + tx]`

- `N[ph*TILE_WIDTH+ty][Col]`

- `N[(ph*TILE_WIDTH+ty)*Width + Col]`

- where ph is the sequence number of the current phase

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {

  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;

 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();

    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();

  }

  P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {

  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;

 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();

    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();

  }

  P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();

  }

  P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
 __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
 __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

 int bx = blockIdx.x;  int by = blockIdx.y;
 int tx = threadIdx.x; int ty = threadIdx.y;

 int Row = by * blockDim.y + ty;
 int Col = bx * blockDim.x + tx;
 float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
   // Collaborative loading of M and N tiles into shared memory
   Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
   Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
   __syncthreads();
   for (int i = 0; i < TILE_WIDTH; ++i)
     Pvalue += Mds[ty][i] * Nds[i][tx];
   __synchthreads();
 }
 P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {

  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();

  }

  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
  // Loop over the M and N tiles required to compute the P element
  for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();
  }
  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Automatic scalar variables: registers

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();

    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();
  }

  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Automatic scalar variables: registers

iterates over phases

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();
  }
  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Automatic scalar variables: registers

iterates over phases

load into shared memory

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
  // Loop over the M and N tiles required to compute the P element
  for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();
  }

  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Automatic scalar variables: registers

iterates over phases

load into shared memory

assures that all threads have loaded the data into shared mem.

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width) {
  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

  int bx = blockIdx.x;  int by = blockIdx.y;
  int tx = threadIdx.x; int ty = threadIdx.y;

  int Row = by * blockDim.y + ty;
  int Col = bx * blockDim.x + tx;
  float Pvalue = 0;
 // Loop over the M and N tiles required to compute the P element
 for (int ph = 0; ph < Width/TILE_WIDTH; ++ph) {
    // Collaborative loading of M and N tiles into shared memory
    Mds[ty][tx] = M[Row*Width + ph*TILE_WIDTH+tx];
    Nds[ty][tx] = N[(ph*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();
    for (int i = 0; i < TILE_WIDTH; ++i)
      Pvalue += Mds[ty][i] * Nds[i][tx];
    __synchthreads();
  }

  P[Row*Width+Col] = Pvalue;
}
```

Shared variables: block scope

Automatic scalar variables: registers

iterates over phases

load into shared memory

assures that all threads have loaded the data into shared mem.

assures that all threads have finished using the data into shared mem.

Tile (Thread Block) Size Considerations

- Each thread block should have many threads
  - TILE_WIDTH of 16 gives 16*16 = 256 threads
  - TILE_WIDTH of 32 gives 32*32 = 1024 threads
- For 16, in each phase, each block performs 2*256 = 512 float loads from global memory for 256 * (2*16) = 8,192 mul/add operations. (16 floating-point operations for each memory load)
- For 32, in each phase, each block performs 2*1024 = 2048 float loads from global memory for 1024 * (2*32) = 65,536 mul/add operations. (32 floating-point operation for each memory load)

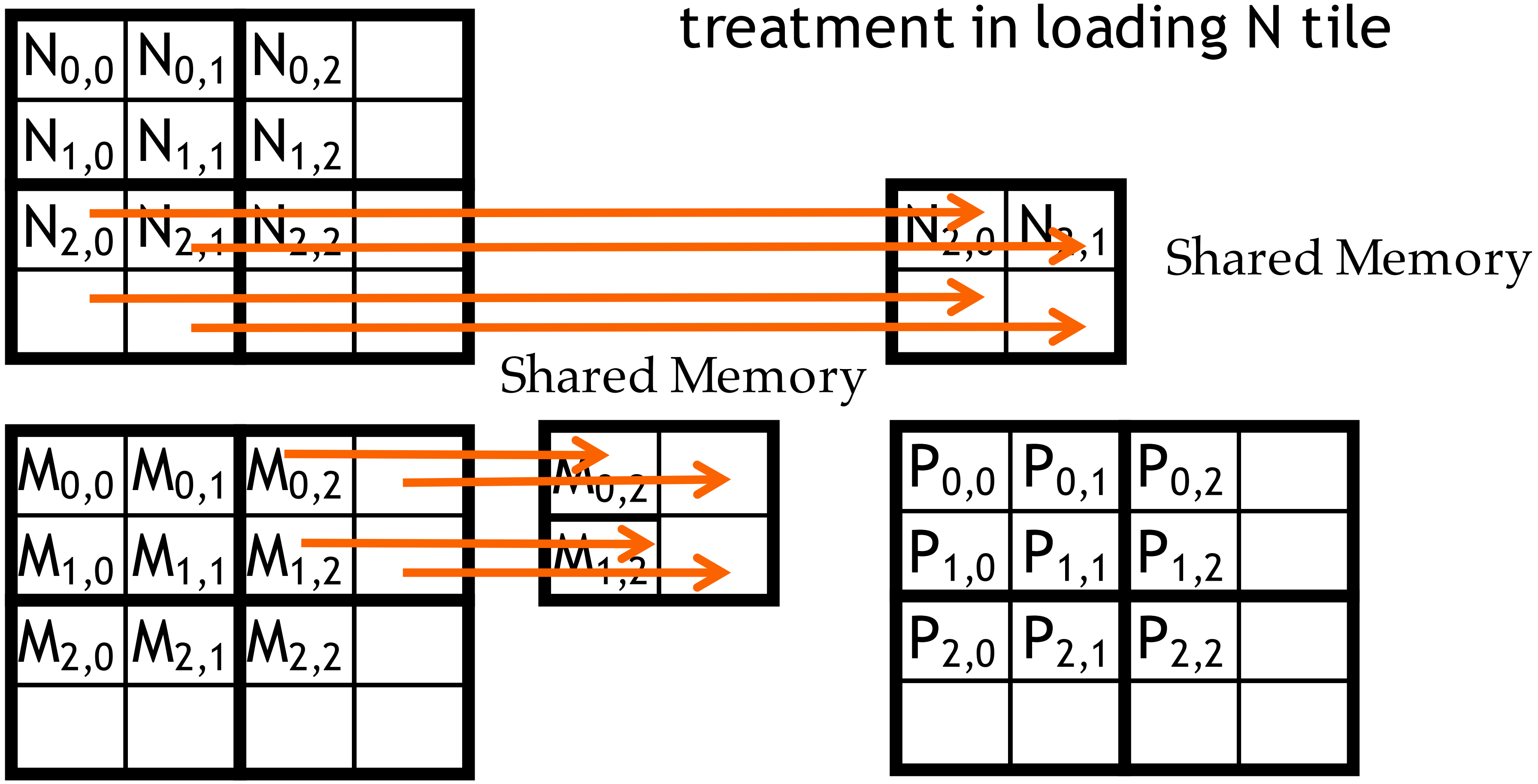
Shared Memory and Threading

- For an SM with 16KB shared memory
  - Shared memory size is implementation dependent!
  - For TILE_WIDTH = 16, each thread block uses 2*256*4B = 2KB of shared memory.
  - For 16KB shared memory, one can potentially have up to 8 thread blocks executing
    - This allows up to 8*512 = 4,096 pending loads. (2 per thread, 256 threads per block)
  - The next TILE_WIDTH 32 would lead to 2*32*32*4 Byte= 8K Byte shared memory usage per thread block, allowing 2 thread blocks active at the same time
  - However, the thread count limitation of 1536 threads per SM in current generation GPUs will reduce the number of blocks per SM to one!
- Each `__syncthread()` can reduce the number of active threads for a block
  - More thread blocks can be advantageous

Boundary checks

- The previous code assumes that the matrix has a size (a width) that is an exact multiple of TILE_WIDTH.
- Let’s extend the code to handle matrices with arbitrary width…
    - …without using padding, that would waste space and time in copying data.

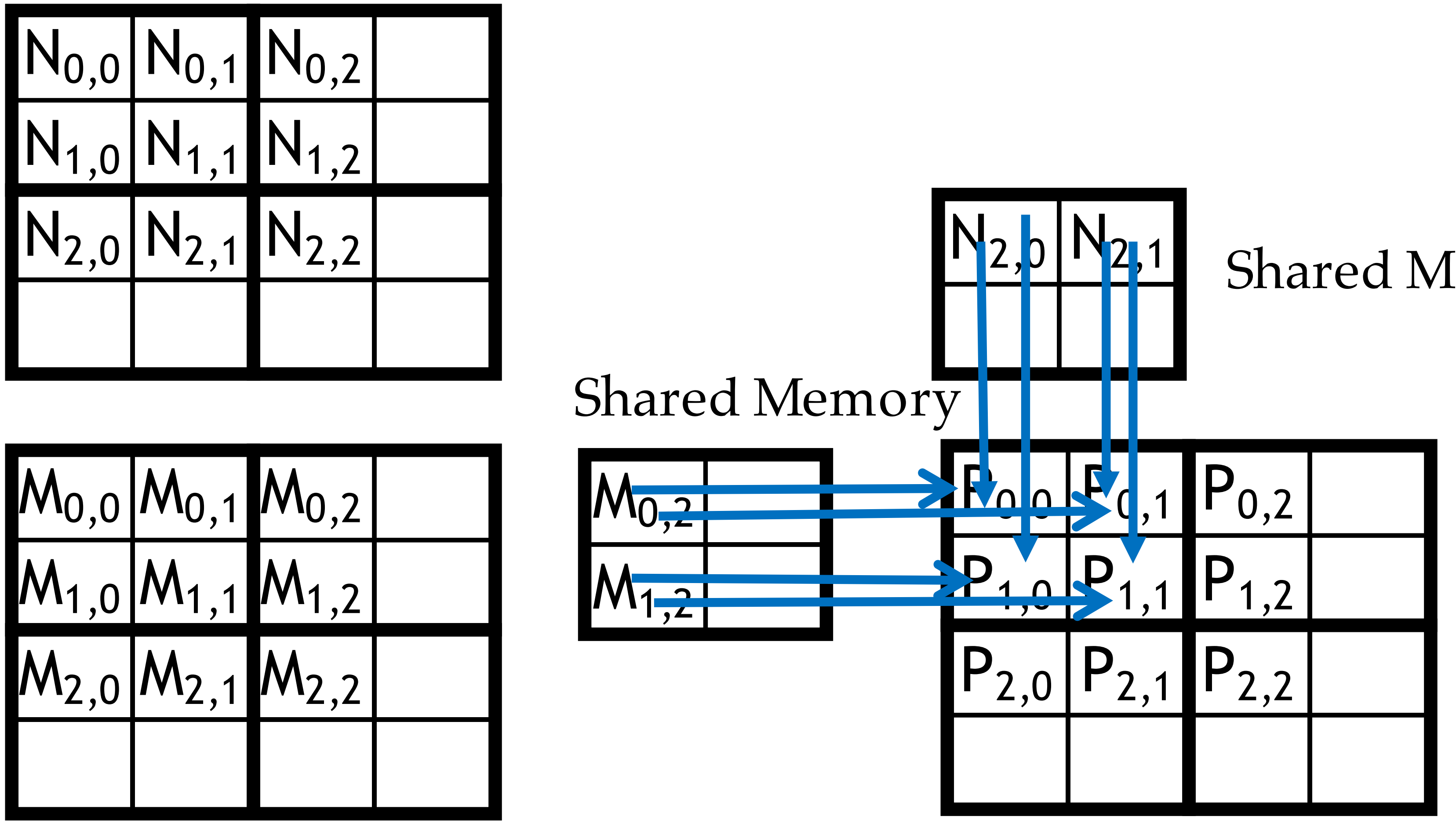
Phase 1 Loads for Block (0,0) for a 3x3 Example



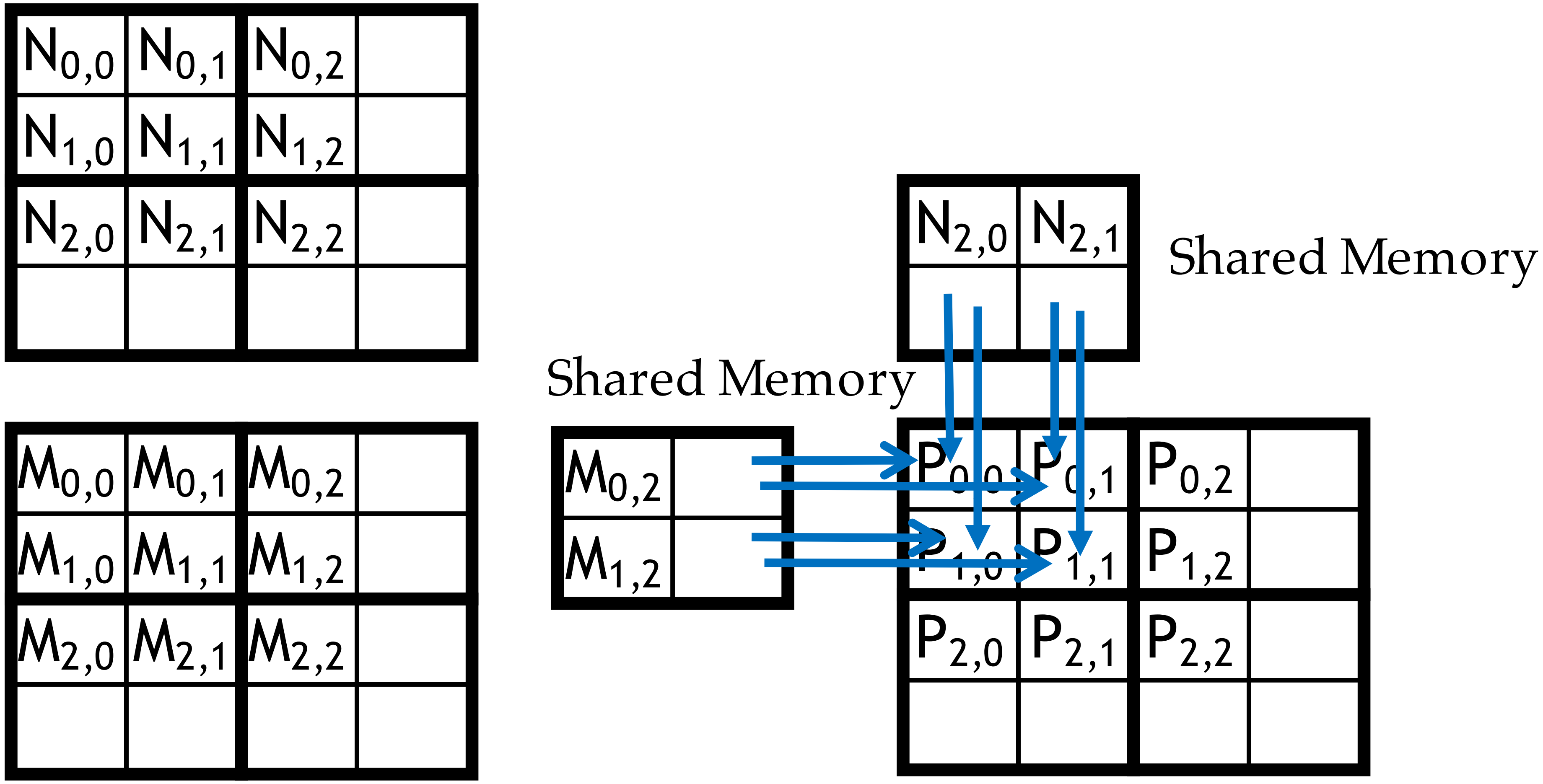
Threads (1,0) and (1,1) need special treatment in loading N tile

Threads (0,1) and (1,1) need special treatment in loading M tile

Phase 1 Use for Block (0,0) (iteration 0)



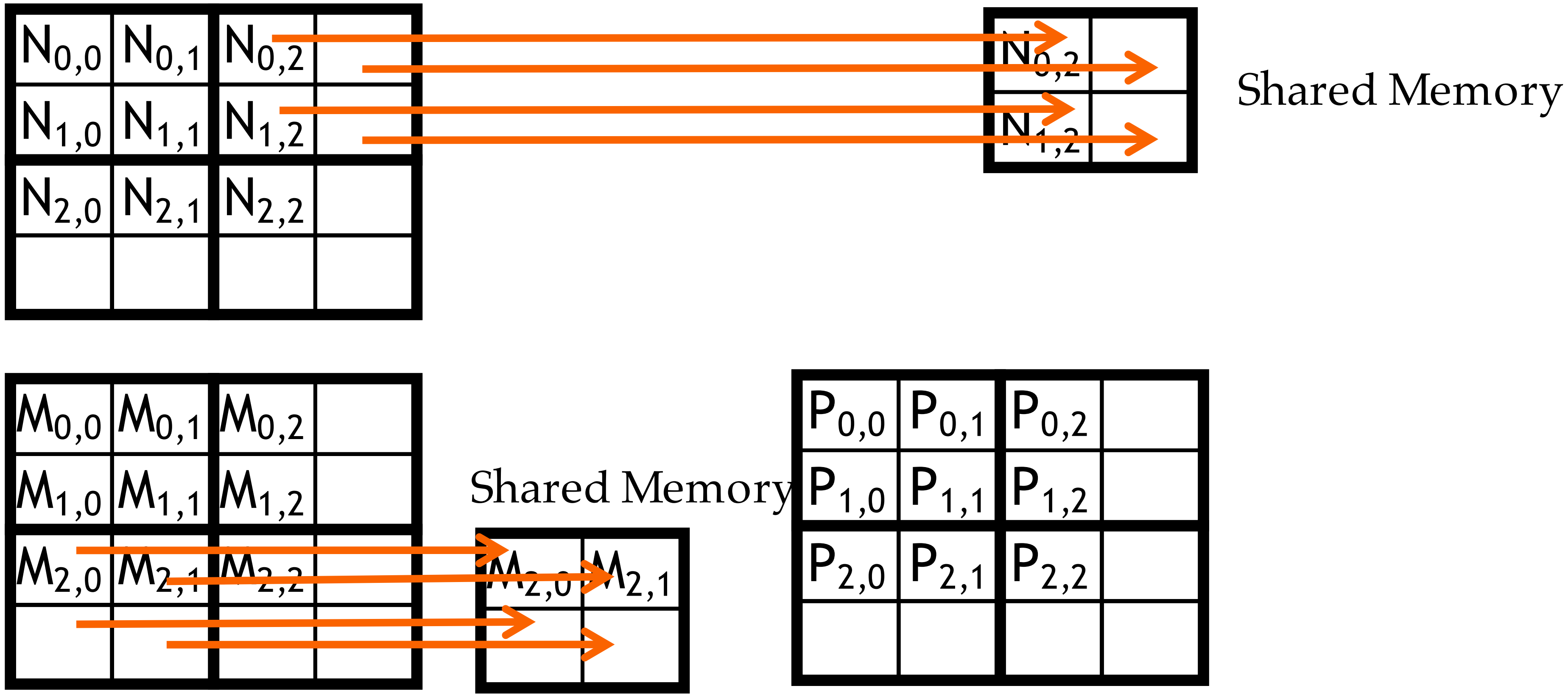
Phase 1 Use for Block (0,0) (iteration 1)


$N_{0,0}$ $N_{0,1}$ $N_{0,2}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$
$M_{0,0}$ $M_{0,1}$ $M_{0,2}$
$M_{1,0}$ $M_{1,1}$ $M_{1,2}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$
$N_{2,0}$ $N_{2,1}$
Shared Memory
Shared Memory
$M_{0,2}$
$M_{1,2}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$

All Threads need special treatment. None of them should introduce invalidate contributions to their P elements.

Phase 0 Loads for Block (1,1) for a 3x3 Example

Threads (0,1) and (1,1) need special treatment in loading N tile

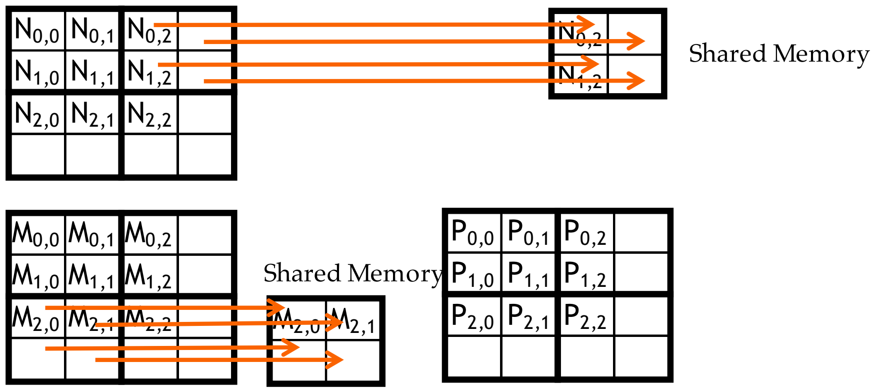

$N_{0,0}$ $N_{0,1}$ $N_{0,2}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$
$N_{0,2}$
$N_{1,2}$
Shared Memory
$M_{0,0}$ $M_{0,1}$ $M_{0,2}$
$M_{1,0}$ $M_{1,1}$ $M_{1,2}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$
Shared Memory
$M_{2,0}$ $M_{2,1}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$

Threads (1,0) and (1,1) need special treatment in loading M tile

Main cases in the example

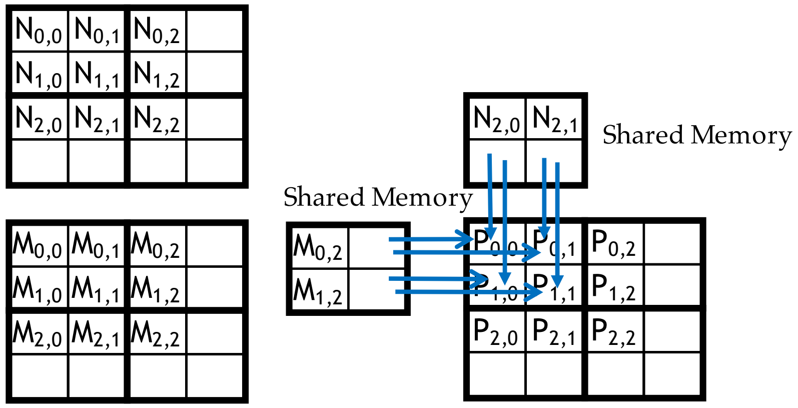
- Threads that do not calculate valid P elements but still need to participate in loading the input tiles
  - Phase 0 of Block(1,1), Thread(0,1), assigned to calculate non-existent P[3,2] but need to participate in loading tile element N[1,2]
- Threads that calculate valid P elements may attempt to load non-existing input elements when loading input tiles
  - Phase 0 of Block(0,0), Thread(1,0), assigned to calculate valid P[1,0] but attempts to load non-existing N[3,0]

Threads (0,1) and (1,1) need special treatment in loading N tile



$N_{0,0}$ $N_{0,1}$ $N_{0,2}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$
$N_{0,2}$
$N_{1,2}$
Shared Memory
$M_{0,0}$ $M_{0,1}$ $M_{0,2}$
$M_{1,0}$ $M_{1,1}$ $M_{1,2}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$
Shared Memory
$M_{2,0}$ $M_{2,1}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$

Threads (1,0) and (1,1) need special treatment in loading M tile



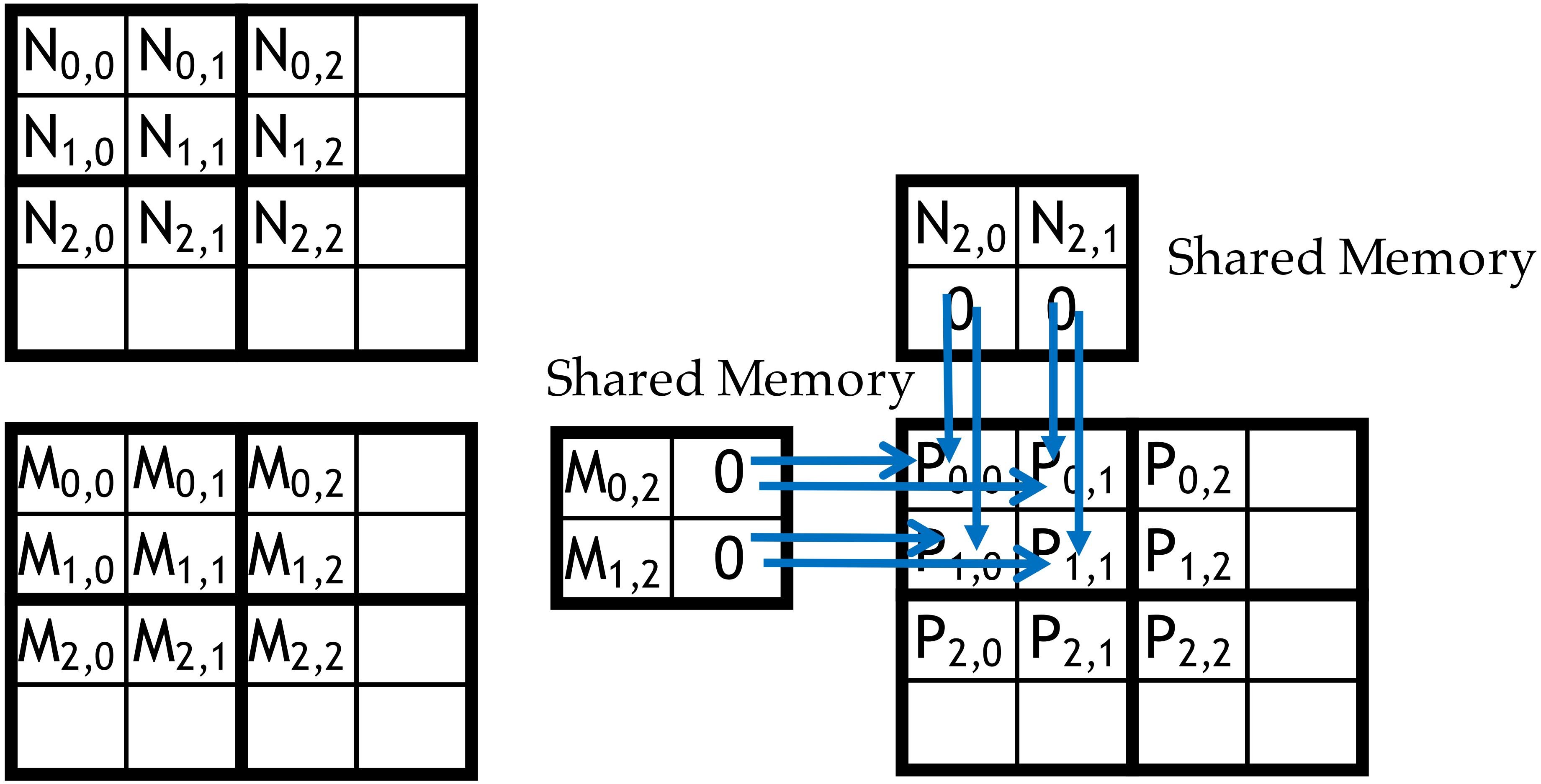
$N_{0,0}$ $N_{0,1}$ $N_{0,2}$
$N_{1,0}$ $N_{1,1}$ $N_{1,2}$
$N_{2,0}$ $N_{2,1}$ $N_{2,2}$
$N_{2,0}$ $N_{2,1}$
Shared Memory
Shared Memory
$M_{0,2}$
$M_{1,2}$
$M_{0,0}$ $M_{0,1}$ $M_{0,2}$
$M_{1,0}$ $M_{1,1}$ $M_{1,2}$
$M_{2,0}$ $M_{2,1}$ $M_{2,2}$
$P_{0,0}$ $P_{0,1}$ $P_{0,2}$
$P_{1,0}$ $P_{1,1}$ $P_{1,2}$
$P_{2,0}$ $P_{2,1}$ $P_{2,2}$

All Threads need special treatment. None of them should introduce invalidate contributions to their P elements.

A solution

- When a thread is to load any input element, test if it is in the valid index range
  - If valid, proceed to load
  - Else, do not load, just write a 0
- Rationale: a 0 value will ensure that that the multiply-add step does not affect the final value of the output element
- The condition tested for loading input elements is different from the test for calculating output P element
  - A thread that does not calculate valid P element can still participate in loading input tile elements

Phase 1 Use for Block (0,0) (iteration 1)



$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$N_{2,0}$	$N_{2,1}$
0	0

Shared Memory

Shared Memory

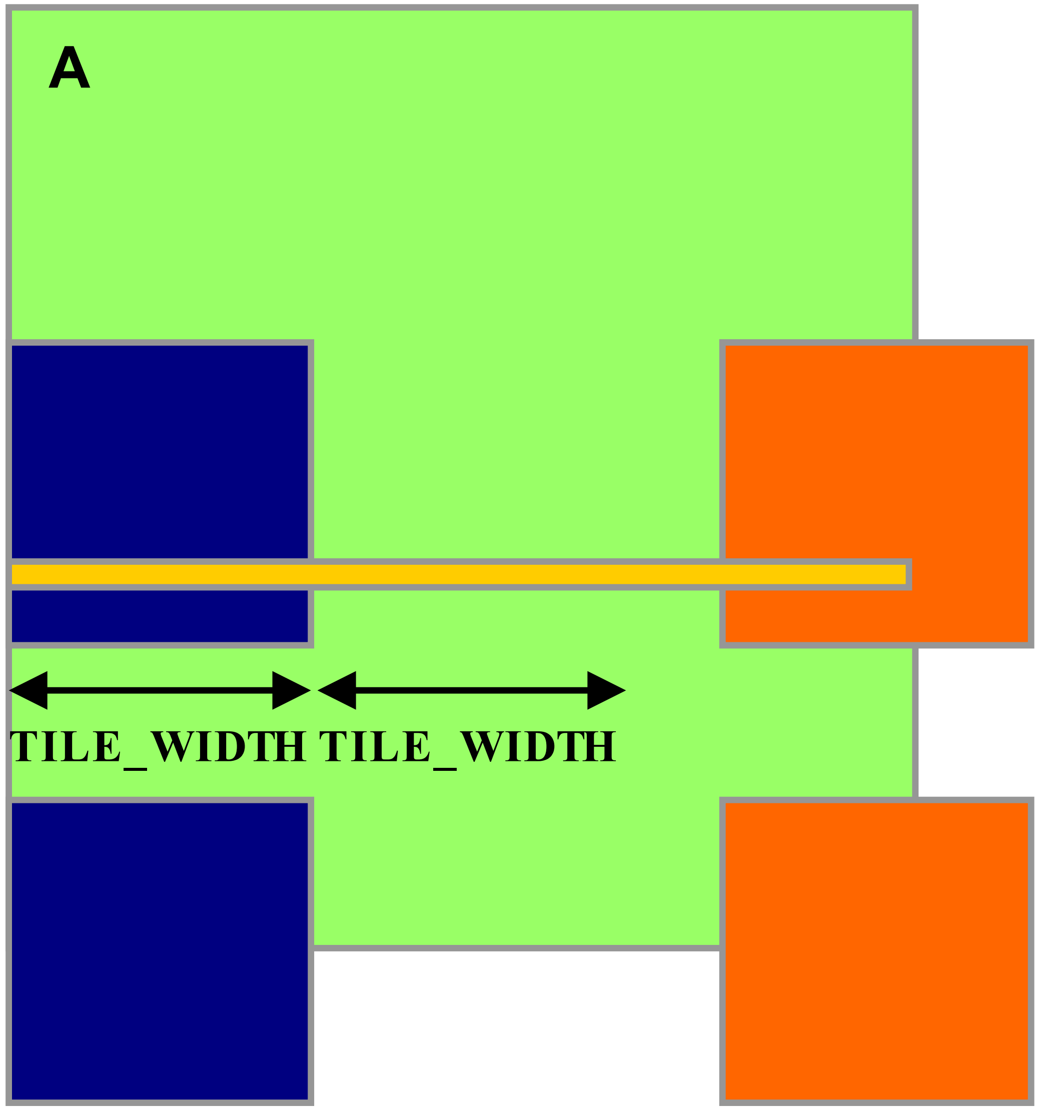
$M_{0,2}$	0
$M_{1,2}$	0

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	

$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	
$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	
$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	

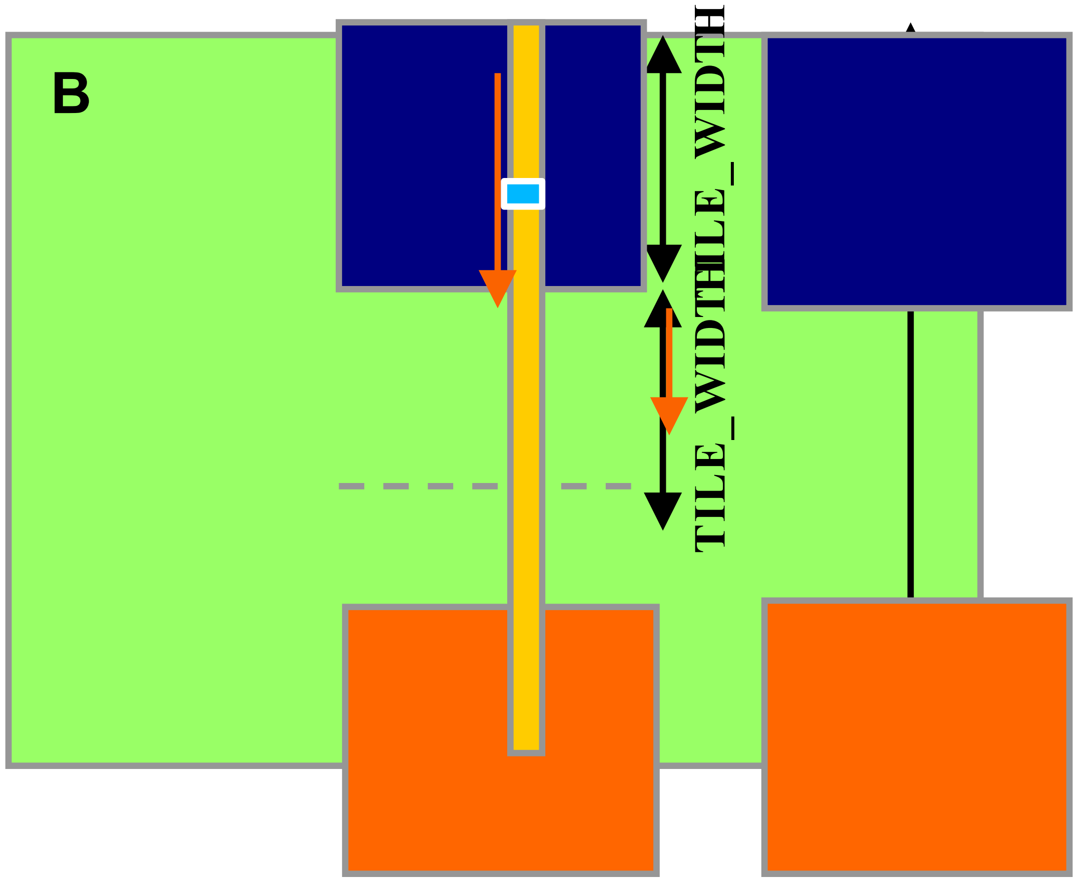
Boundary Condition for Input M Tile

- Each thread loads
  - M[Row][p*TILE_WIDTH+tx]
  - M[Row*Width + p*TILE_WIDTH+tx]
- Need to test
  - (Row < Width) && (p*TILE_WIDTH+tx < Width)
  - If true, load M element
  - Else , load 0



Boundary Condition for Input N Tile

- Each thread loads
  - N[p*TILE_WIDTH+ty][Col]
  - N[(p*TILE_WIDTH+ty)*Width+ Col]
- Need to test
  - (p*TILE_WIDTH+ty < Width) && (Col< Width)
  - If true, load N element
  - Else , load 0



Loading Elements – with boundary check

```
for (int p = 0; p < (Width-1) / TILE_WIDTH + 1; ++p) {
    if(Row < Width && p * TILE_WIDTH + tx < Width) {
        ds_M[ty][tx] = M[Row * Width + p * TILE_WIDTH + tx];
    } else {
        ds_M[ty][tx] = 0.0;
    }
    if (p * TILE_WIDTH + ty < Width && Col < Width) {
        ds_N[ty][tx] = N[(p * TILE_WIDTH + ty) * Width + Col];
    } else {
        ds_N[ty][tx] = 0.0;
    }
    __syncthreads();
}
```

Inner Product – Before and After

```
    if(Row < Width && Col < Width) {

        for (int i = 0; i < TILE_WIDTH; ++i) {

            Pvalue += ds_M[ty][i] * ds_N[i][tx];

        }

        __syncthreads();

    } /* end of outer for loop */

    if (Row < Width && Col < Width)

        P[Row*Width + Col] = Pvalue;

} /* end of kernel */
```

Some Important Points

- For each thread the conditions are different for
  - Loading M element
  - Loading N element
  - Calculating and storing output elements
- The effect of control divergence should be small for large matrices

Handling General Rectangular Matrices

- In general, the matrix multiplication is defined in terms of rectangular matrices
  - A $j \times k$ M matrix multiplied with a $k \times l$ N matrix results in a $j \times l$ P matrix
- So far we have seen square matrix multiplication, a special case
- The kernel function needs to be generalized to handle general rectangular matrices
  - The Width argument is replaced by three arguments: *j*, *k*, *l*
  - When Width is used to refer to the height of M or height of P, replace it with *j*
  - When Width is used to refer to the width of M or height of N, replace it with *k*
  - When Width is used to refer to the width of N or width of P, replace it with *l*

Credits

- These slides report material from:
  - NVIDIA GPU Teaching Kit

Books

- Programming Massively Parallel Processors: A Hands-on Approach, D. B. Kirk and W-M. W. Hwu, Morgan Kaufmann - 2nd edition - Chapt. 4-6

  or

  Programming Massively Parallel Processors: A Hands-on Approach, D. B. Kirk and W-M. W. Hwu, Morgan Kaufmann - 3rd edition - Chapt. 3-5